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Lie algebras in which every 1-dimensional weak subideal is an ideal

Dedicated to the memory of Professor Shigeaki Tôgô

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Introduction

A Lie algebra L is called a $\mathfrak{C}$-algebra if every 1-dimensional subideal of L is an ideal of L and an (A_∞)-algebra if every nilpotent inner derivation of L is zero. In [2] the authors investigated the properties of $\mathfrak{C}$-algebras and related Lie algebras. On the other hand, in [9] Shimizuike and Tôgô investigated the properties of (not necessarily finite-dimensional) (A_∞)-algebras and related Lie algebras. In terms of weak subideals we easily see that a Lie algebra L is an (A_∞)-algebra if and only if every 1-dimensional weak subideal of L is central in L. Thus it seems to be natural to study an intermediate class of Lie algebras between the class of $\mathfrak{C}$-algebras and that of (A_∞)-algebras.

The purpose of this paper is to investigate the property of $\mathfrak{C}(\mathrm{wsi})$-algebras, that is, Lie algebras in which every 1-dimensional weak subideal is an ideal, and to determine the structure of $\mathfrak{C}(\mathrm{wsi})$-algebras under various circumstances.

In Section 2, we shall show that over any field $\mathfrak{C}(\mathrm{wsi})$-algebras belonging to the class $\mathrm{L}(\mathrm{wsi})\acute{\mathrm{E}}(\mathrm{wsi})\mathfrak{A}$ are either abelian or almost-abelian (Theorem 2.2).

In Section 3, we shall show the following results: Let L be a Lie algebra over a field $\mathfrak{k}$ of characteristic zero. If either

(a) L is a serially finite Lie algebra whose locally soluble radical belongs to the class $\acute{\mathrm{E}}(\mathrm{wsi})\mathfrak{A}$, or

(b) L is a subideally finite Lie algebra,

then L is a $\mathfrak{C}(\mathrm{wsi})$-algebra if and only if $L = R \oplus S$, where R is an ideal of L which is either abelian or almost-abelian and S is a semisimple (A_∞)-ideal of L (Theorems 3.3 and 3.8). Moreover, when $\mathfrak{k}$ is an algebraically closed field, if L satisfies either the above statement (a) or

(c) L is a weak-subideally finite Lie algebra,

then L is a $\mathfrak{C}(\mathrm{wsi})$-algebra if and only if L is either abelian or almost-abelian (Theorem 3.3 and Proposition 3.5).

In Section 4, we shall give the following examples over any field;

(i) a $\mathfrak{C}$-algebra which is not a $\mathfrak{C}(\mathrm{wsi})$-algebra (Example 1),

(ii) a $\mathfrak{C}(\mathrm{wsi})$-algebra which is not an (A_∞)-algebra (Example 2),