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# Kirillov-Kostant theory and Feynman path integrals on coadjoint orbits II

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## Introduction

Let $G$ be a connected and simply connected solvable Lie group. In this paper we construct irreducible unitary representations of $G$ by using the Feynman path integrals on coadjoint orbits [1] [13].

In §1, we compute the path integrals on $M = \mathbf{R}^n \times \mathbf{R}^n$. Let $\theta$ be a 1-form on $M$ and $H$ a $C^\infty$-function on $M$ which satisfies certain conditions. The path integral $K_{\theta,H}(x'', x'; T)$ $(x', x'' \in \mathbf{R}^n)$ computed by using the action $\int_0^T \gamma^*\theta - H(\gamma(t))\,dt$ (where $\gamma$ runs over a certain set of paths on $M$) can be written by the solution of differential equations defined by $\theta$ and $H$.

In §2, we investigate the path integrals on coadjoint orbits. Let $\mathfrak{g}$ be the Lie algebra of $G$ and $\mathfrak{g}^*$ the dual space of $\mathfrak{g}$. Fix an element $\lambda$ of $\mathfrak{g}^*$ and choose a real polarization $\mathfrak{p}$. Following the Kirillov-Kostant theory [4] [5] [14], we construct an irreducible unitary representation $\pi_\lambda^{\mathfrak{p}}$ of $G$. We put $\theta_\lambda = \langle \lambda, g^{-1}dg \rangle$ and $H_Y = \langle \lambda, g^{-1}Yg \rangle$ for any $Y \in \mathfrak{g}$. We show that the integral operator of $K_{\theta_\lambda, H_Y}$ corresponds to $\pi_\lambda^{\mathfrak{p}}(exp\,TY)$.

## §1 Path integrals on $\mathbf{R}^n \times \mathbf{R}^n$

In this section, we shall compute the Feynman path integrals on $M = \mathbf{R}^n \times \mathbf{R}^n$. Let $n_1, \dots, n_m$ be natural numbers such that $\sum_{i=1}^m n_i = n$. We put $U^i = \mathbf{R}^{n_i}$ and $V^i = \mathbf{R}^{n_i}$ for $i = 1 \cdots m$. Let ${}^t(x, y) = {}^t(x^1 \cdots x^m y^1 \cdots y^m)$ be the normal coordinates on $M = U^1 \times \cdots \times U^m \times V^1 \times \cdots \times V^m$ where $x^i = {}^t(x^{i,1} \cdots x^{i,n_i}) \in U^i$ and $y^i = {}^t(y^{i,1} \cdots y^{i,n_i}) \in V^i$ for $i = 1 \cdots m$. Let $\theta$ be a 1-form on $M$ and $H$ a $C^\infty$-function on $M$. Suppose that $\theta$ and $H$ are expressed in the following forms respectively:

$$\theta = \sum_{i=1}^m {}^t y^i (dx^i + \sum_{j=1}^{i-1} f^{ij} dx^j) + {}^t a^i\, dx^i + {}^t b^i\, dy^i \tag{1.1}$$

where