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On Support Theorems

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1. Introduction. Let R_n be the n dimensional Euclidean space and let Ξ_n be the dual of R_n. The elements of R_n and Ξ_n are sequences $x=(x_1, x_2, \cdots, x_n)$ and $\xi=(\xi_1, \xi_2, \cdots, \xi_n)$ of real numbers. We put

$$D=(D_1, D_2, \cdots, D_n) \qquad \text{with} \quad D_j=\frac{1}{i}\,\frac{\partial}{\partial x_j} \qquad (j=1, 2, \cdots, n).$$

For convenience' sake we use the notations:

$$x=(x', t), \qquad x'=(x_1, x_2, \cdots, x_{n-1}), \qquad t=x_n,$$

$$\xi=(\xi', \tau), \qquad \xi'=(\xi_1, \xi_2, \cdots, \xi_{n-1}), \qquad \tau=\xi_n,$$

$$D_{x'}=(D_1, D_2, \cdots, D_{n-1}), \qquad D_t=D_n.$$

We denote by Ξ_{n-1} the $n-1$ dimensional space consisting of elements ξ'.

Let $\mathcal{D}$, $\mathcal{S}$ and $\mathcal{O}_M$ be the spaces of all C^∞-functions with compact supports, all rapidly decreasing C^∞-functions and all slowly increasing C^∞-functions on R_n respectively. These spaces are provided with usual topologies of L. Schwartz [4]. Let $\mathcal{D}'$ and $\mathcal{S}'$ be the strong duals of $\mathcal{D}$ and $\mathcal{S}$ respectively and let $\mathcal{O}'_C$ be the space of all rapidly decreasing distributions. We shall denote by $\mathcal{O}_M(\Xi_{n-1})$ the space $\mathcal{O}_M$ considered on Ξ_{n-1}. By the partial Fourier transform of $T \in \mathcal{S}'$ we understand the Fourier transform of T with respect to the first $n-1$ variables which will be denoted by $\hat{T}(\xi', t)$.

For any $A(\xi') \in \mathcal{O}_M(\Xi_{n-1})$, we define the operator $A(D_{x'})$ on $\mathcal{S}'$ as follows: The partial Fourier transform of $A(D_{x'})\,T$, $T \in \mathcal{S}'$, is $A(\xi')\,\hat{T}(\xi', t)$. In this paper we are concerned with the operator of the following form:

$$F(D_{x'}, D_t)=D_t^m+A_1(D_{x'})D_t^{m-1}+\cdots+A_m(D_{x'})$$

$$\text{with} \quad A_j(\xi') \in \mathcal{O}_M(\Xi_{n-1}) \quad (j=1, 2, \cdots, m) \quad \text{and} \quad m \geq 1.$$

J. Peetre observed in [2, 3] that the operator

$$D_t-i(1+\sum_{j=1}^{n-1}D_j^2)^{1/2}$$