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Axiomatic Treatment of Full-superharmonic Functions

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Introduction.

There is an axiomatic theory of harmonic functions or an axiomatic potential theory, developed by M. Brelot for the most part and investigated further by others. (See [5] for a bibliography.) The starting point of this theory is the introduction of a sheaf of functions on a locally compact space satisfying certain axioms (see [3], [4] and [5] for details). These axioms are abstracted from the characteristic properties of harmonic functions in the classical potential theory on a Euclidean space or on a Riemann surface. Thus the sheaf is called a harmonic structure.

On the other hand, a notion of full-superharmonic functions on a Riemann surface was introduced by Z. Kuramochi [9] and thoroughly investigated by C. Constantinescu and A. Cornea [6]. (Also, see [11] and [13].) The theory of full-superharmonic functions is, for the most part, quite parallel to that of superharmonic functions in the classical potential theory. Therefore, the axiomatic theory by Brelot, which gives a methodology to the classical theory of superharmonic functions, is readily extended to an axiomatic theory of full-superharmonic functions, once a suitable additional structure is given. In this paper, we shall show how this extension is carried out.

There are many variations in axioms to be assumed for the harmonic structure. In this paper, we choose Axioms T and H, which are Axioms 2 and 3 of Brelot ([3], [4] or [5]). We introduce an additional structure in §2 and assume two axioms (Axioms S and $\tilde{T}$) for it. The structure thus given will be called a full-harmonic structure. Besides the one on a Riemann surface introduced by Kuramochi, we have examples of full-harmonic structure defined for solutions of second order elliptic partial differential equations.

From this full-harmonic structure, we construct a theory of full-superharmonic functions. We follow the author's previous paper [11] for the construction of the theory, while we apply Brelot's methods to the proofs. Definitions and properties of full-superharmonic functions are discussed in §3 and §4. In particular, §4 is devoted to the study of full-superharmonic functions of potential type. We shall call them $\mathcal{P}$-functions. In Kuramochi's theory on Riemann surfaces, a kernel (Green function) for the full-harmonic structure is introduced and integral representation of $\mathcal{P}$-functions with respect to this kernel is discussed. (See [6], [9] [11] and [13]; the kernel is