J. Sci. Hiroshima Univ. Ser. A-I
31 (1967), 13-15

A Remark on Vector Fields on Lens Spaces

Toshio Yoshida

(Received February 17, 1967)

§1. Introduction

Let M be a C^∞-manifold. The (continuous) vector field v on M is a cross-section of the tangent bundle of M, and k-field on M is a set of k vector fields $v_1, \cdots, v_k$ such that the k vectors $v_1(x), \cdots, v_k(x)$ are linearly independent for each point $x \in M$. We denote by $span(M)$ the maximal number of k where M admits a k-field.

In this note, it is remarked that $span(L^n(p))$, of the $(2n+1)$-dimensional mod p lens space $L^n(p)$, is given partially by the following

Proposition. *Let* $n+1=m2^t$ (m: odd), $t+1=c+4d$ $(0\leqq c\leqq 3)$

(i) *If* $c=0$, *then* $2t+1\leqq span(L^n(p))\leqq 2t+2$ $(=span(S^{2n+1}))$.

(ii) *If* $c=1, 2$, *then* $span(L^n(p))=2t+1$ $(=span(S^{2n+1}))$.

(iii) *If* $c=3$, *then* $2t+1\leqq span(L^n(p))\leqq 2t+3$ $(=span(S^{2n+1}))$.

Here the lens space $L^n(p)$ $(p>1)$ is the quotient space S^{2n+1}/Γ of the unit sphere S^{2n+1} by the topological transformation group $\Gamma=\{1, \gamma, \cdots, \gamma^{p-1}\}$ defined by

$$\gamma\cdot(z_0, z_1, \cdots, z_n)=(e^{2\pi i/p}z_0, e^{2\pi i/p}z_1, \cdots, e^{2\pi i/p}z_n)$$
$$((z_0, z_1, \cdots, z_n)\in S^{2n+1}\subset C^{n+1}).$$

We notice that the above proposition holds in the following form for the case $p=2$:

$$span(L^n(2))=span(S^{2n+1}).$$

This follows easily from the fact that $L^n(2)$ is the $(2n+1)$-dimensional real projective space RP^{2n+1}, and

$$span(RP^n)=span(S^n),$$

which is an immediate consequence of the fact that S^n has a linear k-field, $k=span(S^n)$.

Also, we notice that there is a lens space such that

$$span(L^n(p))<span(S^{2n+1}),$$