J. Sci. Hiroshima Univ. Ser. A-I
31 (1967), 151-160

On the Torsion Submodule of a Module of Type (F_1)

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(Received September 16, 1967)

Introduction

In the paper [3] E. Kunz has proved the following:

Let R be an integral domain with quotient field K, S be a subring of R and k be its quotient field. If the module of S-differentials of R is finitely generated and if the module of k-differentials of k has rank r, then the $(r-1)$-th Kähler different of R over S vanishes and the r-th Kähler different does not.

In connection with this fact we introduce the following notion. A finitely generated module (over a commutative ring with unity element) is said to be a module of type (F_r) if its $(r-1)$-th Fitting ideal (i.e. Determinantenideal in [2]) is the zero ideal and its r-th Fitting ideal is a regular ideal.

The purpose of this paper is mainly to study the torsion submodule of a module of type (F_1). In §1 we give the definition of a module of type (F_r) and state some properties of modules of this type. In §2 we study the torsion submodule of a module of type (F_1), and in §3 we prove that, for a noetherian domain R of Krull dimension one, an R-module of type (F_1) is the direct sum of its torsion submodule and a free module of rank one if, and only if, its dual module is a free module of rank one. In §4 we apply the results of the preceding sections to the module of differentials on an affine curve defined over a perfect field.

Throughout this paper, all rings will be assumed to be commutative with unity element and all modules to be unitary.

The writer wishes to express his thanks to Professor Y. Nakai, who has often helped him with discussion out of which the ideas in this work developed.

§1. The module of type (F_r)

Let R be a ring and M be a finitely generated R-module. For a system $\{x_1, \cdots, x_n\}$ of generators of M there is an exact sequence

$$(1) \qquad 0 \longrightarrow N \longrightarrow R^n \xrightarrow{\varphi} M \longrightarrow 0,$$

where R^n is a free R-module with a system $\{e_1, \cdots, e_n\}$ of basis, the R-homomorphism φ is defined by $\varphi(e_j)=x_j$ and N is the kernel of φ. Let N be gener-