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On Kuramochi's Function-theoretic Separative Metrics on Riemann Surfaces

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Introduction

In order to extend Fatou's and Beurling's theorems to arbitrary Riemann surfaces, Z. Kuramochi introduced ([4]; also see [5] and [7]) two notions of function-theoretic separative metrics, i.e., H. B. and H. D. separative metrics.

Since extended Fatou's and Beurling's theorems are stated in terms of compactifications of an open Riemann surface, we shall define separative compactifications rather than separative metrics. In this paper we shall give necessary and sufficient conditions for a compactification to be H. B. or H. D. separative, in terms of the Wiener or the Royden compactification, respectively. Our characterizations are given in a simple form compared with the original definition by Z. Kuramochi and may make it easier to comprehend the meaning of these notions.

In §1, we shall discuss compactifications of a hyperbolic Riemann surface R. §2 (resp. §3) is devoted to the study of harmonic measures (resp. capacitary potentials) which were defined by Z. Kuramochi ([3]). We shall investigate their properties on the Wiener or the Royden boundary of R. In §4 (resp. §5), we shall give our main theorems on H. B. (resp. H. D.) separative compactifications and study the relation between H. B. and H. D. separative compactifications (§5).

As an application, we shall show in §6: 1) for Fatou's theorem, Kuramochi's result ([4], [5], [7]) and Constantinescu and Cornea's result (Satz 14.4 in [2]) are equivalent; 2) for Beurling's theorem, Kuramochi's result ([4], [5], [7]) is independent of a similar result by Constantinescu and Cornea (Satz 18.1 in [2]).

Notation and terminology

Let R be a hyperbolic Riemann surface. For a subset A of R, we denote by ∂A and A^i the (relative) boundary and the interior of A respectively. We shall call a closed subset F of R *regular* if ∂F consists of at most a countable number of analytic arcs clustering nowhere in R.

An *exhaustion* will mean an increasing sequence $\{R_n\}_{n=1}^{\infty}$ of relatively