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On the Existence of Solutions of Some Non-linear Parabolic Equations

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1. Introduction

In this paper we consider parabolic equations with boundary conditions:

$$\text{(a)} \quad \frac{du}{dt}+Au=f, \qquad u(0)=u_0,$$

$$\text{(b)} \quad \frac{du}{dt}+Au=f, \qquad u(0)=u(T),$$

where A is a non-linear operator.

In 1965 J. Leray and J. L. Lions [4] introduced a non-linear operator on a reflexive Banach space into its conjugate space and showed that it is surjective under the condition of coerciveness. Making use of this result, J. L. Lions [5] showed the existence of solutions of (a) and (b) for a certain kind of non-linear operator A.

In 1968 H. Brezis [1] introduced a new operator, called of type M, which is more general than the operator of J. Leray and J. L. Lions, and showed that the operator of type M on a reflexive Banach space into its conjugate space is also surjective under the condition of coerciveness.

The purpose of this paper is to extend J. L. Lions' results in [5] on the existence of solutions of (a) and (b) to the case where A is a bounded coercive operator satisfying conditions which are more general than Lions' [5]. In the proof we shall make use of the result by H. Brezis mentioned above.

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2. Notation and statement of theorems

In general, for a Banach space U over C (complex numbers), we shall denote the anti-dual space of U by U'. Let H be a Hilbert space over C, $(\ ,\ )$ be the scalar product in H, and $|\cdot|$ be the norm in H. One may identify H' with H. Let V be a reflexive Banach space over C, $((\ ,\ ))$ the natural pairing between V' and V, $\|v\|_V$ the norm of $v \in V$ and $\|v^*\|_{V'}$ the norm of $v^* \in V'$.

Assume that $V \subset H$, V is dense in H and the injection is continuous. Then $V \subset H \subset V'$. Let F be a linear space whose elements are vector-valued