J. Sci. HIROSHIMA UNIV. SER. A-I
**34** (1970), 259-270

# *Explicit and Implicit Difference Formulas of Higher Order Accuracy for One-dimensional Heat Equation*

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(Received September 16, 1970)

## 1. Introduction

In this paper we are concerned with the first and the second boundary value problems for the one-dimensional heat equation

$$u_t(x, t) = u_{xx}(x, t) \qquad (0 \leqq x \leqq 1,\ 0 \leqq t) \tag{1.1}$$

with the initial condition

$$u(x, 0) = \varphi(x) \qquad (0 \leqq x \leqq 1). \tag{1.2}$$

For the numerical solution of this problem by the finite-difference methods, there are known the two-level explicit formula with the truncation error of order $h^2$, Crank-Nicolson's method [16][1], Douglas' high order correct method [4], three-level difference formulas [6], and so on.

The object of this paper is to construct two-level explicit formulas with truncation errors of orders $h^4$ and $h^6$, to determine their ranges of stability, and to derive the unconditionally stable two-level implicit formulas of higher order accuracy. Although the formulas obtained here are not all new, the stability conditions are considered in a somewhat unified form. These formulas will be useful not only for the direct use but also for the approximation of the truncation errors of the formulas of the lower order accuracy.

## 2. Preliminaries

### 2.1 Difference formulas

Let $h$ and $k$ be the mesh-sizes in the $x$- and $t$-directions respectively and put $r = k/h^2$. Then, for the function $u(x, t)$ which is sufficiently smooth and satisfies the equation (1.1), using the relations

$$\frac{\partial^{2n} u}{\partial x^{2n}} = \frac{\partial^n u}{\partial t^n} \qquad (n = 1, 2, \cdots), \tag{2.1}$$

1) Numbers in square brackets refer to the references listed at the end of this paper.