

# *Asymptotic Expansions of Some Test Criteria for Homogeneity of Variances and Covariance Matrices From Normal Populations*

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**0. Introduction and summary.** It is very difficult to obtain exact distributions of most of the test statistics used in the multivariate analysis. Therefore, some statisticians have hitherto obtained asymptotic expansion of test criteria instead of the exact distributions. This paper is divided into five parts and deals with the problem of asymptotic expansions of test criteria on variances and covariances in normal populations. In Part I we compare a modified likelihood ratio test (Bartlett [3]) with the asymptotically UMP invariant test (Lehmann [13]) for testing homogeneity of variances of  $k$