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Note on the Canonical Extensions and the Boundary Values for Distributions in the Space H^μ

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In our previous paper [3], the multiplicative product between distributions was investigated together with the related topics. With the viewpoints mentioned there in mind, in this paper we shall study the problems centering around the notions of the trace, the section, the boundary value and the canonical extension, for distributions especially in H^μ. The present paper is in a sense a continuation of our related paper [2].

The general discussions about these notions are made in Section 1 with reference especially to the canonical extension. Sections 2 and 3 are devoted to the discussions about the trace mapping and the canonical extension for distributions in the space H^μ and we have tried to make clear the close relationship between them. Some complements to our previous paper [2] are given with new results. In the final section, the notions of $\mathscr{S}'$-boundary value and $\mathscr{S}'$-canonical extension are introduced and discussed. We can speak of $\mathscr{D}'_{L^2}$-boundary value and $\mathscr{D}'_{L^2}$-canonical extension and so on. However, we do not proceed to the study about these matters, because the treatment involves no essential difficulty, of course, though it is necessary to introduce modifications into our considerations given in this section in order to obtain the analogues.

1. Preliminaries

We first recall some notions concerning multiplicative product (or simply product) between distributions closely connected with the discussions in the subsequent sections. Let $u, v \in \mathscr{D}'(R_N)$, where R_N is an N-dimensional Euclidean space. If the distributional limit $\lim_{j\to\infty}(u*\rho_j)v$ exists for any δ-sequence $\{\rho_j\}$, the limit is uniquely determined, which is called the product in the strict sense and denoted by $u\cdot v$. We have shown in [9, p. 225] that if the limit exists, then $\lim_{j\to\infty} u(v*\rho_j)$ exists and

$$(1) \qquad \lim_{j\to\infty}(u*\rho_j)v=\lim_{j\to\infty}u(v*\rho_j)$$

for any δ-sequence $\{\rho_j\}$. The above definition with $\{\rho_j\}$ replaced by $\{\phi_\varepsilon\}$, $\varepsilon>0$,