HIROSHIMA MATH. J.
2 (1972), 271–283

On the Ring of Endomorphisms of an Indecomposable Injective Module over a Prüfer Ring

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(Received September 15, 1972)

It is well known, in the theory of abelian groups, that the ring of endomorphisms of the p-torsion part of Q/Z is the p-adic integers; in modern language, the p-torsion part of Q/Z is nothing but the injective envelope of the additive group Z/p. The aim of this paper is to study the ring of endomorphisms of indecomposable injective modules over Prüfer rings, which generalizes the above fact.

In §1, we shall deal with a basic theory of indecomposable injective modules over commutative rings and this enables us to reduce the problem to the case where valuation rings act as operators. The notion of pseudo-convergence plays an essential role in our discussions; it is originally due to Ostrowsky and by employing it, *I.* Kaplansky succeeded in proving the uniqueness of a maximal immediate extension under some conditions. In §2, the relationship between the module structure and the breadth of a pseudo-cenvergent set is cleared up. In §5, it is shown that the ring of endomorphisms of an indecomposable injective module over a Prüfer ring is, rather unexpectedly, not commutative and the structure of its center is determined.

Throughout this paper a ring R will always be understood to be commutative, to have a unit and a module over R to be unitary.

§1. Preliminaries

We say that an R-module M is co-irreducible if M is not zero and has no non-zero submodules $N_i (i=1, 2)$ such that $N_1 \cap N_2 = 0$. A submodule N of an R-module M is irreducible in M if the quotient module M/N is co-irreducible; this is equivalent to saying that N is properly contained in M and if N is the intersection of submodules N_1 and N_2, then $N_1 = N$ or $N_2 = N$. It is clear that non-zero submodules and essential extensions of a co-irreducible module are co-irreducible. Therefore we see immediately that the injective envelope $E(M)$ of a co-irreducible module M is co-irreducible and the order ideal $0(x)$ of every non-zero element x of $E(M)$ is an irreducible ideal of R.

We denote by Σ the set of irreducible ideals in R. Let I be a member of

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1) $0(x)$ means the ideal annihilating the element x.