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Nonlinear Operators of Monotone Type in Reflexive Banach Spaces and Nonlinear Perturbations

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Introduction

In this paper we are concerned with nonlinear operators of monotone type from a reflexive Banach space X into the dual space X^*. Such operators have been considered to make a general treatment of boundary value problems for nonlinear elliptic partial differential equations and initial-boundary value problems for nonlinear parabolic partial differential equations. Studies of nonlinear operators of monotone type have been made by many authors (e.g., [1]–[3], [5]–[7], [9]–[12], [15]–[18], [20], [22], [25]).

In [2] Brezis introduced two classes of nonlinear singlevalued operators, called of type M and pseudo-monotone respectively, from X into X^* and then established existence theorems for nonlinear functional equations of the forms

$$(a) \quad Ax=f \qquad \text{for given } f \in X^*$$

and

$$(b) \quad Ax+Tx=f \qquad \text{for given } f \in X^*,$$

where A is an operator of type M or a pseudo-monotone operator from X into X^* and T is a nonlinear monotone operator from X into X^*. Recently, the concept of pseudo-monotone operators was generalized by Browder and Hess [10] to the multivalued case. Many results in [2] on the solvability of (a) and (b) were extended to the multivalued case where the equations have the forms:

$$(a)' \quad Ax \ni f \qquad \text{for given } f \in X^*$$

and

$$(b)' \quad Ax+Tx \ni f \qquad \text{for given } f \in X^*.$$

In this paper we shall first give a natural generalization of the notion of operators of type M to the multivalued case, and investigate basic properties of such operators. Next, we shall solve nonlinear equations of types $(a)'$ and $(b)'$ for multivalued operators of type M and multivalued pseudo-monotone operators under somewhat different assumptions from those in [2], [10], [11] and [22].