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# *Some Remarks on the Cauchy Problem for p-Parabolic Equations*

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In his paper [11] S. Mizohata gave a semi-group theoretic treatment of the Cauchy problem for a regularly $p$-parabolic equation. This was successfully done with the aid of an operator matrix $H_q(t)=H_q(x, t, D_x)$ introduced therein. Recently D. Ellis [2] developed a Hilbert space approach to the Cauchy problem for a uniformly $p$-parabolic equation, following in rough outline the method explored by S. Kaplan [9] in his treatment of the Cauchy problem for a parabolic operator $\frac{\partial}{\partial t}-L(t)$, where $L(t)$ is uniformly strongly elliptic. Generally, in such an approach, special attention has been paid to find out energy estimates appropriate to the problem. As for the Cauchy problem for a specified parabolic system (§ 6 in [7]), the present author, in collaboration with K. Yoshida, has tried a generalization of Kaplan's treatment indicated above by introducing a certain type of energy estimates.

The main purpose of this paper is to investigate the uniqueness and existence theorems of a solution to the Cauchy problem for a regularly $p$-parabolic equation from a Hilbert space approach as done by D. Ellis [2], relying upon another type of energy estimates which will be established with the aid of a prescribed operator matrix $H_q(t)$, and following the same arguments as in our treatment (§ 6 in [7]) of a parabolic system.

By the Cauchy problem we shall always mean a fine Cauchy problem as described in paper [7]. With this in mind, in Section 1, some notations and functional spaces are introduced with a precise formulation of such a Cauchy problem for a regularly $p$-parabolic equation, where the notions of the $\mathscr{D}'_{L^2}$-boundary value and the $\mathscr{D}'_{L^2}$-canonical extension of a distribution are discussed in some detail. In Section 2 the energy inequalities (cf. Theorems 1 and 2 below) for a regularly $p$-parabolic operator and for its dual operator are derived by making use of the operator matrix $H_q(t)$, which was introduced by S. Mizohata [11]. The former estimate will be of a type very similar to the one obtained in [7, Theorem 8]. These estimates enable us to apply a Hilbert space approach to our problem. Finally in Section 3 the uniqueness and existence theorems for our problem are discussed along this line of thought. We improve some of the results obtained by D. Ellis [2]. Combining Corollary 4 with Proposition 5 below, we have a