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An Integral Representation of an Eigenfunction of Invariant Differential Operators on a Symmetric Space

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1. Introduction

Let G be a connected semisimple Lie group with finite center and K a maximal compact subgroup of G. In [6], Harish-Chandra determined the Plancherel measure for the symmetric space G/K. The spherical Fourier transform may be regarded as a method of representing a more or less arbitrary spherical function as a linear combination of elementary spherical functions. On the other hand Ehrenpreis proved in [3], [4] and [5] that for various spaces W of functions or distributions on $\boldsymbol{R}^n$ (such as the space of solutions of linear constant coefficient partial differential equations) any $T \in W$ admits a representation

$$T(x) = \int \exp i < z, x > d\mu(z)/a(z)$$

where μ is a bounded measure on a "multiplicity variety", a is an element of an "analytic uniform structure" for W, and where the integral converges in a certain sense. Now, since elementary spherical functions are eigenfunctions of all invariant differential operators on the symmetric space G/K, the above result of Ehrenpreis suggests an analogous problem of representing an eigenfunction of a system of invariant differential operators as an integral of those elementary spherical functions which satisfy the same system of invariant differential equations. In this paper, we shall give a solution to this problem.

The authors are grateful to Professor L. Ehrenpreis for helpful discussions. He also raised the problem of extending the result of this paper to K-infinite eigenfunctions by using matrix coefficients of the principal series representations. We shall deal with this problem in the forthcoming paper.

2. Notation

We denote by $C^\infty(G)$ the space of all C^∞ functions on G with its usual topology. Let $\mathfrak{g}$ be the Lie algebra of all left invariant vector fields on G and let $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ be a Cartan decomposition where $\mathfrak{k}$ is the Lie algebra of K. If $x = k \exp X$ ($k \in$