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Oscillation Criteria for a Second Order Differential Equation with a Damping Term

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1. Introduction

In this paper we are concerned with the oscillatory behavior of the second order differential equation with a damping term

$$x''+q(t)x'+p(t)f(x)=0 \tag{1}$$

where the following assumptions are assumed to hold:

(a) $p, q \in C(R^+)$, $R^+=(0, \infty)$;

(b) $f \in C(R)$, $R=(-\infty, \infty)$, and $xf(x)>0$ for all $x \in R-\{0\}$;

(c) $f \in C^1(R-\{0\})$, and there is a constant $k>0$ such that $f'(x) \geqq k$ for all $x \in R-\{0\}$.

We restrict our attention to solutions $x(t)$ of (1) which exist on some half-line $[T_x, \infty)$ and are nontrivial for all large t. A solution $x(t)$ of (1) is said to be oscillatory if $x(t)$ has an unbounded set of zeros $\{t_k\}_{k=1}^{\infty}$ such that $\lim_{k\to\infty} t_k=\infty$; otherwise, a solution is said to be nonoscillatory. Equation (1) is called oscillatory (or nonoscillatory) if all solutions of (1) are oscillatory (or nonoscillatory).

As a special case of (1) we have

$$x''+p(t)f(x)=0, \tag{2}$$

which has been the subject of intensive investigations since the pioneering work of Atkinson [1]. For results regarding oscillation of (2) with the assumption $p(t) \geqq 0$ we refer in particular to Wong [14]. Oscillation criteria for (2) with no sign assumption on $p(t)$ have been given by Waltman [12], Bhatia [3], Kiguradze [9], Kamenev [7], Staikos and Sficas [11] and others. Recently an attempt has been made by Erbe [6] to extend to (1) some of the known results for (2).

It is the object of this paper to present oscillation criteria for equation (1) with no explicit sign assumptions on $p(t)$ and $q(t)$. Our results do not overlap with those of Erbe.

Results for (1) with nonlinear damping have been obtained by Bobisud ([4],