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Asymptotic Properties of Solutions of Two-dimensional Differential Systems with Deviating Argument

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Introduction

In this paper we consider the two-dimensional differential system with deviating argument

$$\text{(A)} \qquad \begin{cases} x'(t) = p(t)y(t) \\ y'(t) = f(t, x(g(t))) \end{cases}$$

which, in the particular case where $p(t)>0$, is equivalent to the second order scalar differential equation

$$\text{(B)} \qquad \left(\frac{1}{p(t)} x'(t)\right)' = f(t, x(g(t))).$$

The conditions we always assume for p, g, f are as follows:

(a) $p(t)$ is continuous and nonnegative on $[a, \infty)$; $p(t) \not\equiv 0$ on any infinite subinterval of $[a, \infty)$.

(b) $g(t)$ is continuous on $[a, \infty)$ and $\lim_{t\to\infty} g(t) = \infty$.

(c) $f(t, x)$ is continuous on $[a, \infty)\times(-\infty, \infty)$ and $|f(t, x)| \leqq \omega(t, |x|)$ for $(t, x) \in [a, \infty)\times(-\infty, \infty)$ where $\omega(t, r)$ is continuous on $[a, \infty)\times[0, \infty)$ and nondecreasing in r.

We note that $g(t)$ is a general deviating argument, that is, it is allowed to be *retarded* ($g(t) \leqq t$) or *advanced* ($g(t) \geqq t$) or otherwise. System (A) is called *superlinear* or *sublinear* according to whether $\omega(t, r)/r$ is nondecreasing or nonincreasing in r for $r>0$.

The purpose of this paper is to study the asymptotic behavior of solutions of system (A) which is either superlinear or sublinear. We are particularly interested in obtaining information about the growth or decay of oscillatory solutions as well as of nonoscillatory solutions. Hereafter the term "solution" will be understood to mean a solution $\{x(t), y(t)\}$ of (A) which exists on some half-line $[\tau, \infty)$, $\tau > a$, and satisfies

$$\sup\{|x(t)| + |y(t)| : t \geqq \tau'\} > 0 \qquad \text{for any} \quad \tau' \geqq \tau.$$