HIROSHIMA MATH. J.
12 (1982), 103–113

Dirichlet problem for a semi-linearly perturbed structure of a harmonic space

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(Received August 31, 1981)

Introduction

In [3], the author considered a semi-linear perturbation of a harmonic space and discussed Dirichlet problems of Perron-Brelot type with respect to the perturbed structure. In the present note, we further investigate such Dirichlet problems. In § 2, we are concerned with the problem whether a bounded boundary function, which is resolutive with respect to the original structure, remains resolutive with respect to the perturbed structure. Then, in § 3, we give sufficient conditions for a boundary point to be regular with respect to the Dirichlet problem for the perturbed structure. The results in § 3 are extensions of those in [2] where linear perturbations are treated.

As a simple but typical example to which our theory can be applied, consider a semi-linear equation

$$(*) \qquad \Delta u = q(x)\psi(u)$$

on a domain $\Omega \subset \mathbf{R}^n$ ($n \geqq 2$; Ω: hyperbolic if $n=2$), where q is a non-negative function belonging to $L^r_{\mathrm{loc}}(\Omega)$ with $r > n/2$ and ψ is a non-decreasing locally Lipschitz-continuous function on $\mathbf{R}$ such that $\psi(t_0)=0$ for some $t_0 \in \mathbf{R}$. For a compactification Ω^* of Ω and a bounded function φ on $\Omega^* \setminus \Omega$ which is resolutive with respect to $\Delta u = 0$, our theorems in § 2 imply the following results:

(i) Without any further assumptions on ψ, if $\varphi \geqq t_0$ or $\varphi \leqq t_0$, then φ is resolutive with respect to $(*)$;

(ii) If either ψ^+ or ψ^- is convex, then φ is always resolutive with respect to $(*)$.

As to regularity, our results in § 3 show that $\xi \in \Omega^* \setminus \Omega$ is regular with respect to the Dirichlet problem for $(*)$ if it is regular for $\Delta u = 0$ and if there exist an open neighborhood V of ξ in Ω^* and a potential p on $V \cap \Omega$ such that $p(x) \to 0$ as $x \to \xi$ and $\Delta p = -q$ on $V \cap \Omega$. Note that these conditions do not refer to the function ψ.

§ 1. Notation and basic assumptions

Let $(X, \mathscr{U})$ be a harmonic space in the sense of Constantinescu-Cornea [1]