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On the distribution of a statistic in multivariate inverse regression analysis

Yasunori FUJIKOSHI and Ryuei NISHII

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§ 1. Introduction

In a multivariate inverse regression problem we are interested in making inference about an unknown $q \times 1$ vector $\boldsymbol{x}=(x_1,\dots, x_q)'$ from an observed $p \times 1$ response vector $\boldsymbol{y}=(y_1,\dots, y_p)'$. Brown [3] has summarized various aspects of the problem. We assume that $\boldsymbol{y}$ is random, $\boldsymbol{x}$ is fixed and

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{a} + B'\boldsymbol{x} + \boldsymbol{e} \\ &= \Theta' \begin{bmatrix} 1 \\ \boldsymbol{x} \end{bmatrix} + \boldsymbol{e} \end{aligned} \tag{1.1}$$

where $\Theta'=[\boldsymbol{a}, B']$: $p \times (1+q)$ is the matrix of unknown parameters and $\boldsymbol{e}$ is an error vector having a multivariate normal distribution $N_p[\boldsymbol{0}, \Sigma]$. Further, suppose that the N informative observations on $\boldsymbol{y}$ and $\boldsymbol{x}$ have been given. When $p \geq q$, it is possible to obtain a natural point estimate for $\boldsymbol{x}$, and to construct a confidence region for $\boldsymbol{x}$, based on a statistic, which is a quadratic form of the estimate. For an application of the confidence region it is required to give the upper percentage point of the statistic.

The purpose of this paper is to study the distribution of the statistic mentioned above. We shall derive an asymptotic expansion for the distribution function of the statistic up to the order N^{-2} and hence for the upper percentage point of the statistic. In Section 3 we treat the distribution problem in the situation where Θ is known and Σ is unknown. We note that the distribution of the statistic in this case is essentially the same as one of a statistic in growth curve model. The distribution has been studied by Rao [6] and Gleser and Olkin [4]. The numerical accuracy of our asymptotic approximations is checked by comparing with exact results of Gleser and Olkin [4]. In Section 4 we treat the distribution problem in the situation where Θ and Σ are unknown. In this case a reduction of the distribution problem is given. By using the reduction and perturbation method we shall obtain the asymptotic expansion of the distribution function of the statistic. Some formulas used in deriving the asymptotic expansions are summarized in Section 5.