

## Pseudo-coalescent classes of groups

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### Introduction

Since Wielandt's fundamental paper [19] appeared, subnormality and coalescence in groups have been studied by many group-theorists, Baer, Gruenberg, Lennox, Robinson, Roseblade, Stonehewer and others in the last twenty years. In [18] Tôgô introduced two concepts of weak subnormality and weak ascendancy which are generalizations of subnormality and ascendancy. By using these concepts, we shall introduce the concepts of pseudo-coalescence, ascendant pseudo-coalescence, local pseudo-coalescence and locally ascendant pseudo-coalescence which are corresponding to coalescence, ascendant coalescence, local coalescence and locally ascendant coalescence respectively, and also the analogies of those in Lie algebras. We call a class  $\mathfrak{X}$  of groups pseudo-coalescent (resp. ascendantly pseudo-coalescent) if in any group the join of any pair of a subnormal (resp. an ascendant)  $\mathfrak{X}$ -subgroup and a weakly subnormal (resp. a weakly ascendant)  $\mathfrak{X}$ -subgroup is always a weakly subnormal (resp. a weakly ascendant)  $\mathfrak{X}$ -subgroup. We also call a class  $\mathfrak{X}$  of groups locally pseudo-coalescent (resp. locally ascendantly pseudo-coalescent) if whenever  $H$  is a subnormal (resp. an ascendant)  $\mathfrak{X}$ -subgroup and  $K$  is a weakly subnormal (resp. a weakly ascendant)  $\mathfrak{X}$ -subgroup of a group  $G$  every finite subset  $F$  of  $J = \langle H, K \rangle$  is contained in some weakly subnormal (resp. weakly ascendant)  $\mathfrak{X}$ -subgroup  $X$  of  $G$  with  $X \leq J$ . Among the known coalescent classes are those of finite groups (Wielandt [19]), groups satisfying the maximal condition for subgroups (Baer [3]), finitely generated nilpotent groups (Baer [2]), groups satisfying the minimal condition for subnormal subgroups (Robinson [9] and Roseblade [12]), groups satisfying the maximal condition for subnormal subgroups (Roseblade [13]) and any subjunctive class of finitely generated groups (Roseblade and Stonehewer [14]). It is known that the following classes are ascendantly coalescent:  $\{Q, E\}$ -closed classes of groups satisfying the minimal condition for subnormal subgroups (Robinson [9]),  $\{N_0, S\}$ -closed classes of groups satisfying the maximal condition for subgroups (cf. Robinson [10]) and certain classes of finitely generated groups, e.g., the classes of finitely generated groups satisfying the maximal condition for ascendant subgroups (Hulse [5]). Among the known locally coalescent classes are the class of nilpotent groups (Baer [2]) and any subjunctive class (Roseblade and Stonehewer [14]). However, very little is known concerning locally ascendant