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Explicit two-step methods with one off-step node

Hisayoshi SHINTANI

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1. Introduction

Consider the initial value problem

$$y' = f(x, y), \quad y(x_0) = y_0, \tag{1.1}$$

where $f(x, y)$ is assumed to be sufficiently smooth. Let $y(x)$ be the solution of (1.1) and

$$x_n = x_0 + nh \qquad (n=1, 2,\dots; h>0), \tag{1.2}$$

where h is a stepsize. Let y_1 be an approximation of $y(x_1)$ obtained by some appropriate method. We are concerned with the case where the approximations y_j $(j=2, 3,\dots)$ of $y(x_j)$ are obtained by two-step methods. Conventional two-step methods such as linear two-step methods [1], pseudo-Runge-Kutta methods [1, 3] and so on [4] require starting values y_0 and y_1 to generate y_j $(j=2, 3,\dots)$.

In our previous paper [4] we introduced a set of subsidiary nodes

$$x_{n+v} = x_0 + (n+v)h \quad (n=0, 1,\dots; 0<v<1) \tag{1.3}$$

and at the cost of providing an additional starting value y_v we proposed two-step methods for computing y_{n+1} $(n=1, 2,\dots)$ together with subsidiary approximations y_{n+v} of $y(x_{n+v})$, which are of the form

$$y_{n+v} = y_n + b_{r-1}(y_n - y_{n-1}) + d_{r-1}(y_n - y_{n-1+v}) + h\textstyle\sum_{j=0}^{r-1} c_{r-1j}k_{jn}, \tag{1.4}$$

$$y_{n+1} = y_n + b_r(y_n - y_{n-1}) + h\textstyle\sum_{j=0}^{r} c_{rj}k_{jn}, \tag{1.5}$$

where

$$k_{0n} = k_{2n-1}, \quad k_{1n} = f(x_{n-1+v}, y_{n-1+v}), \quad k_{2n} = f(x_n, y_n), \tag{1.6}$$

$$k_{in} = f(x_n + a_i h, y_n + b_i(y_n - y_{n-1}) + d_i(y_n - y_{n-1+v}) + h\textstyle\sum_{j=0}^{i-1} c_{ij}k_{jn}), \tag{1.7}$$

$$a_i = b_i + (1-v)d_i + \textstyle\sum_{j=0}^{i-1} c_{ij}, \quad 0 < a_i \leqq 1 \quad (3 \leqq i \leqq r), \tag{1.8}$$

and a_i, b_i, d_i, and c_{ij} $(j=0, 1,\dots, i-1; i=3, 4,\dots, r)$ are real constants. It has been shown that for $r=4$, 5 there exist a method (1.5) of order $r+1$ and a method (1.4) of order r with $r-2$ function evaluations per step.