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Spherical hyperfunctions on the tangent space of symmetric spaces

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Introduction

Let G be connected semisimple Lie group, σ an involutive automorphism of G and H an open subgroup of fixed points of σ. Then G/H is called a semisimple symmetric space and the tangent space at the origin of G/H is identified with a complement $\mathfrak{q}$ of $\mathfrak{h}$ in $\mathfrak{g}$, where $\mathfrak{g}$ and $\mathfrak{h}$ are the Lie algebras coresponding to G and H, respectively.

In this paper, we consider spherical hyperfunctions on $\mathfrak{q}$ that are H-invariant and simultaneously eigen hyperfunctions on $\mathfrak{q}$. There have appeared several papers dealing with spherical functions on $\mathfrak{q}$ ([1], [2], [3], [5], [9], [10]). In his paper [2], van Dijk listed up spherical distributions for the rank 1 case. On the other hand, in his paper [1], Cerezo determined the dimension of $O(p, q)$ (or $SO_0(p, q)$) invariant spherical hyperfunctions on $\boldsymbol{R}^{p+q}$, where $\boldsymbol{R}^{p+q}$ can be regarded as the tangent space of the semisimple symmetric space; $SO_0(p+1, q)/SO_0(p, q)$. However, studying spherical hyperfunctions, the author found interesting phenomenon. That is; if f is an H-invariant eigen hyperfunction then f is $\tilde{H}$-invariant, where $\tilde{H}$ is the connected component of the Lie group of all non-singular transformations T on $\mathfrak{q}$ such that $p(Tx) = p(x)$ for any H-invariant polynomial p and $x \in \mathfrak{q}$. In fact, $\tilde{H}$ is "large" (if $G = SL(m+1, \boldsymbol{R})$ and $H = GL^+(m, \boldsymbol{R})$, then $\dim H = m^2$ and $\dim \tilde{H} = 2m^2 - m$). It seems that this phenomenon is independent of the category of functions but is dependent on H or $\tilde{H}$ orbits structure on $\mathfrak{q}$. In his paper [8], Ochiai deals with this problem as $\mathscr{D}$-module structure generated by the Lie algebra $\mathfrak{h}$ or $\tilde{\mathfrak{h}}$ which is the Lie algebra corresponding to $\tilde{H}$.

In this paper, we prove that for "generic" eigen values if f is an H-invariant eigen hyperfunction then f is $\tilde{H}$-invariant (see Theorem 5.1 in §5). From Cerezo's result and Theorem 5.1, we can determine the dimension of spherical hyperfunctions on $\mathfrak{q}$ when rank $\mathfrak{q} = 1$ and eigen value $\mu \neq 0$ (see §5).