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Allowable delays for positive diffusion processes

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1. Introduction Many diffusion processes are well modelled by parabolic equations of the form

$$u_t = a(t)u_{xx} - p(x, t)u + q(x, t)u \tag{1.1}$$

where a, p, and q are nonnegative coefficients representing phenomena which underlie the diffusion process. For example, in population dynamics the term au_{xx} corresponds to diffusion due to local concentration, while $-pu$ and qu correspond to death and birth rates, respectively.

Since such phenomena may not lead to instantaneous changes in population size, it is natural to include delays in the models under consideration. Thus in problems of population dynamics, chemical reactions, etc., it is important to be able to generalize (1.1) to delay parabolic equations of the form

$$u_t = a(t)u_{xx} - p(x, t)u(x, t-\sigma) + q(x, t)u(x, t-\rho) \tag{1.2}$$

where the delays σ and ρ are nonnegative constants.

The values to be assigned to such delays will depend largely on an understanding of the mechanics of the diffusion process itself. However, in many situations an additional constraint arises from the fact that the solution of the diffusion process is inherently positive. The purpose of the present paper is to establish upper bounds on delays which result from the requirement that $u(x, t) > 0$ for $t \geq 0$.

In case the coefficients of (1.2) are constants such conditions can sometimes be obtained by assuming solutions of the form $u(x, t) = e^{\lambda t}e^{\mu x}$, leading to the characteristic equation

$$\lambda = a\mu^2 - pe^{-\lambda\sigma} + qe^{-\lambda\rho}. \tag{1.3}$$

If (1.2) is accompanied by boundary conditions such as

$$u(0, t) - \alpha u_x(0, t) = 0$$
$$u(L, t) + \beta u_x(L, t) = 0$$

with α, $\beta \geq 0$, then it follows that $\mu^2 \leq 0$ and the absence of real solutions to

$$\lambda + pe^{-\lambda\sigma} \leq qe^{-\lambda\rho} \tag{1.4}$$