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On strongly exact sequences of cocommutative Hopf algebras

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An exact sequence

$$\text{(S)} \qquad k \longrightarrow G \longrightarrow H \longrightarrow J \longrightarrow k$$

of cocommutative Hopf algebras over a field k is said to be strongly exact if for any cocommutative coalgebra C the induced sequence

$$\text{(CS)} \quad e \longrightarrow \mathrm{Hom}_{coal}(C, G) \longrightarrow \mathrm{Hom}_{coal}(C, H) \longrightarrow \mathrm{Hom}_{coal}(C, J) \longrightarrow e$$

of groups is exact. In [6] we gave several equivalent conditions for (S) to be strongly exact in case H is irreducible (i.e., hyperalgebra).

Recently, Yanagihara has shown that when H is pointed, (S) is strongly exact if and only if the sequence

$$(\mathrm{S}^1) \qquad k \longrightarrow G^1 \longrightarrow H^1 \longrightarrow J^1 \longrightarrow k$$

of irreducible Hopf algebras extracted from (S) is strongly exact ([9], Theorem 2).

The main purpose of this paper is to generalize these results.

When H is irreducible, we showed in [6] that one of necessary and sufficient conditions for (S) to be strongly exact is that the Hopf subalgebra G has a coalgebra retraction in H, that is, there exists a coalgebra homomorphism η of H into G such that $\eta|_G = \mathrm{id}_G$. This is valid for cocommutative pointed Hopf algebras ([9], Theorem 2). But, generally, this is not sufficient. In fact, we show in Section 2 that we must demand G to have not only a coalgebra retraction but also a G-linear coalgebra retraction (Theorem 2.7 (3)).

When H is a cocommutative pointed Hopf algebra over k, the structure of H is completely determined by those of its irreducible component H^1 containing 1 and coradical H_0 ([7], §8.1). They are considered in Sections 3 and 4. In Section 3 we show that if a sequence (S) is strongly exact then so is the sequence (S^1) (Theorem 3.5). In Section 4 we prove that the coradical H_0 of a cocommutative Hopf algebra H over k is a Hopf subalgebra if and only if the dual algebra of H_0 is a direct product of separable extension fields of k (Theorem 4.7). In this case we show that if (S) is strongly exact then so is the sequence