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On oscillations of neutral equations with mixed arguments

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1. Introduction and preliminaries

Consider the neutral differential equation

$$\frac{d}{dt}[y(t) + py(t-\tau)] + qy(t-\sigma) = 0 \tag{1}$$

where p, τ, q and σ are real numbers. The main results in this paper are the following:

THEOREM 1. *The following statements are equivalent*:

(a) *Every bounded solution of Eq.* (1) *oscillates.*

(b) *The characteristic equation associated with Eq.* (1)

$$F(\lambda) = \lambda + \lambda p e^{-\lambda\tau} + q e^{-\lambda\sigma} = 0 \tag{2}$$

has no roots in $(-\infty, 0]$.

THEOREM 2. *The following statements are equivalent*:

(a) *Every unbounded solution of Eq.* (1) *oscillates.*

(b) *The characteristic equation* (2) *associated with Eq.* (1) *has no roots in* $(0, \infty)$ *and* 0 *is not a double root of Eq.* (2).

An immediate corollary of the above theorems is the following result which was proved in [3].

COROLLARY *Every solution ofEq.* (1) *oscillates if and only if its characteristic equation* (2) *has no real roots.*

As is customary a solution of Eq. (1) is called *oscillatory* if it has arbitrarily large zeros. Otherwise it is called *nonoscillatory.*

In the sequel all functional inequalities that we write are assumed to hold eventually, that is for sufficiently large t.