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A note on Picard principle for rotationally invariant density

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A nonnegative locally Hölder continuous function P on the punctured closed unit disk $0 < |z| \leq 1$ will be referred to as a *density* on $\Omega: 0 < |z| < 1$. For a density P on Ω we consider the Martin compactification Ω_P^* of Ω with respect to the equation

$$(1) \qquad L_P u \equiv \Delta u - Pu = 0 \qquad (\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2)$$

on Ω. We say that the Picard principle is valid for P if the set of Martin minimal boundary points over the origin $z = 0$ consists of a single point. In the case that P is a *rotationally invariant* density on Ω, i.e., a density P satisfying $P(z) = P(|z|)$ $(z \in \Omega)$, the Martin compactification Ω_P^* is characterized completely by Nakai [3] in terms of what he calls the singularity index $\alpha(P)$ of P at $z = 0$:

$$\Omega_P^* \simeq \{\alpha(P) \leq |z| \leq 1\} ;$$

in particular, the Picard principle is valid for P if and only if $\alpha(P) = 0$.

Take two sequences $\{a_n\}$ and $\{b_n\}$ $(n = 1, 2, \cdots)$ in the interval $(0, 1)$ satisfying $b_{n+1} < a_n < b_n$ with $\{a_n\}$ tending to zero as $n \to \infty$. We consider a sequence of annuli:

$$A_n \equiv \{z \in \boldsymbol{C}: a_n \leq |z| \leq b_n\} , \qquad A = \textstyle\bigcup_{n=1}^{\infty} A_n ,$$

in Ω and set

$$P_n \equiv (2\pi)^{-1} \iint_{A_n} P(z)\, dx\, dy + 1 .$$

The purpose of this note is to show the following

THEOREM. *Let P be a rotationally invariant density. If sequences $\{a_n\}$, $\{b_n\}$, and $\{P_n\}$ satisfy the condition*

$$(2) \qquad \sum_{n=1}^{\infty} \frac{\{\log (b_n/a_n)\}^2}{1 + P_n \log (b_n/a_n)} = +\infty ,$$

then the Picard principle is valid for P at $z = 0$.

COROLLARY 1. *If sequences $\{a_n\}$ and $\{b_n\}$ satisfy the conditions*