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Asymptotic distributions of the MLE's and the LR test in the growth curve model with a serial covariance structure

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1. Introduction

In the growth curve model of Potthoff and Roy [10] we observe an $N \times p$ random matrix Y whose rows are independently distributed as $N_p(\cdot, \Sigma)$ with

$$E[Y] = A\Xi B, \tag{1.1}$$

where A and B are known $N \times k$ and $q \times p$ design matrices of ranks k and $q \leq p$, respectively, and Ξ is a $k \times q$ matrix of unknown parameters. This model is also called a GMANOVA model since the model in the special case of $B = I_p$ is a MANOVA model. The model in the case when Σ is arbitrary positive definite has been studied by many authors. A comprehensive review is given by Grizzle and Allen [5].

In this paper we consider the case when Σ has a serial covariance structure, or an autoregressive structure of the first order

$$\Sigma = \sigma^2 G(\rho) = \sigma^2(\rho^{|i-j|}), \qquad i, j = 1, 2, \ldots, p, \tag{1.2}$$

where $\sigma > 0$ and $|\rho| < 1$ are unknown. In most applications of the model (1.1), p is the number of time points observed each of the N subjects, $(q-1)$ is the degree of polynomial, and k is the number of groups. Further, p is small. For the situations, it is natural to assume (1.2) as a covariance structure. In fact, Lee [8] has pointed out that the serial covariance structure (1.2) is appropriate for three sets of real data. Fujikoshi, Kanda and Tanimura [4] studied the limiting distributions of the MLE(maximum likelihood estimate)'s of ρ and σ^2 and the LR(likelihood ratio) test for (1.2) in the situation where p and k are fixed and $N \to \infty$. The purpose of this paper is to extend the limiting results by finding the next terms in the asymptotic expansions. Some preliminary results on our asymptotic method are given in Section 2. In Section 3 we obtain an asymptotic expansion of the distribution of the MLE's of ρ and σ^2 up to the order $N^{-1/2}$. In Section 4 we discuss with refinements of chi-square approximation to the null distribution of LR statistic for (1.2).