HIROSHIMA MATH. J.
20 (1990), 477–513

Fibred Riemannian spaces with quasi Sasakian structure

Byung Hak KIM

(Received September 13, 1989)

Introduction

Recently Y. Tashiro and the present author [35] have studied fibred Riemannian spaces with almost Hermitian or almost contact metric structure, and given its applications to tangent bundles of Riemannian spaces. In [23] we have also studied fibred Riemannian spaces with vanishing contact Bochner curvature tensor and constructed an example of such spaces which is not a Sasakian space form.

As the first step, it is natural to consider fibred Riemannian spaces with invariant fibres normal to the structure vector. Such a space does not admit nearly Sasakian or contact structure but a quasi Sasakian or cosymplectic structure. This is a motivation for our study of fibred Riemannian spaces with quasi Sasakian or cosymplectic structure.

The notion of quasi Sasakian structure on an almost contact metric manifold was first introduced by D. E. Blair [2] and its properties have been studied by himself, J. C. Gonzalez and D. Chinea [16], S. Kanemaki [19, 20], J. A. Oubina [28] and S. Tanno [33]. It is known that a quasi Sasakian manifold with $d\eta = 0$ or $2\Phi = d\eta$ is cosymplectic or Sasakian, respectively, and there is no quasi Sasakian structure of even rank [2].

An almost contact metric manifold, the structure tensor ϕ of which is Killing, is called a nearly cosymplectic manifold, which was introduced by D. E. Blair [3]. A five-dimensional sphere S^5 admits a nearly cosymplectic structure but not a cosymplectic structure. Besides, M. Capursi [7], D. Chinea and C. Gonzalez [8, 9], I. Goldberg and K. Yano [15], Z. Olszak [26] and J. Oubina [28] have introduced new classes of almost contact metric structures as generalizations of cosymplectic structure. Essential examples of the various structures are given in the papers cited above.

On the other hand, cosymplectic space forms were studied by S. S. Eum [13], S. Kanemaki [20] and G. D. Ludden [24]. S. S. Eum [13] defined the cosymplectic Bochner curvature tensor and it vanishes in a cosymplectic space form. K. Yano [36, 37] considered complex conformal or contact conformal connection to give sufficient conditions in order that the Bochner or contact Bochner curvature tensor vanishes. Similar studies were made by T. Kashiwada