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On the differential equation $y'' + p(t)|y'|\,\mathrm{sgn}\,y + q(t)y = 0$

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0. Introduction. Let $y = y(t)$ be a nontrivial solution of the differential equation

$$(0.1) \qquad y'' + p(t)y' + q(t)y = 0$$

on the interval I and $t_1, t_2 \in I$ be consecutive zeros of $y(t)$ such that $t_1 < t_2$ and $|p(t)| \le M_1$, $|q(t)| \le M_2$ for $t_1 \le t \le t_2$. Then the well-known inequality of de la Vallée Poussin [16] states that for $h = t_2 - t_1$ the relation

$$\frac{1}{2}M_2h^2 + 2M_1h > 1$$

holds. There were several attempts to sharpen this inequality (see for references in [10] pp. 375–376) and Z. Opial [12] has established the optimal inequality of this form

$$(0.2) \qquad M_2h^2 + 2M_1h \ge \pi^2.$$

Recently J.H.E. Cohn [2] has found another inequality

$$(0.3) \qquad h \ge 2\int_0^\infty \frac{ds}{M_2s^2 + M_1s + 1}.$$

We shall see that this inequality is sharper than (0.2). The Cohn's proof is a skillful application of some differential inequlity which has the flavour of a particular Sturmian comparison theorem. Just this is the direction in which we shall proceed in this paper to obtain (0.3) and similar results.

It is well-known that Sturm [15] worked out his theorems for differential equations of the self-adjoint form

$$(0.4) \qquad (r(t)y')' + q(t)y = 0$$

(see [5] or [6]). Kamke [7] gave a new proof of Sturmian theorems by using Prüfer transformation and his method made possible the extension of the Sturmian theorems to half-linear second order differential equations of the form

$$(0.5) \qquad (r(t)y')' + q(t)f(y, r(t)y') = 0$$