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Saddlepoint approximation for the distribution function of the mean of random variables

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§ 1. Introduction

After Daniels [1] introduced a saddlepoint technique in statistics, this method is widely discussed for deriving an accurate approximation for the probability density function of the mean of a random sample. Reid [6] gave an excellent review in this field, and Davison and Hinkley [3] presented a non-parametric saddlepoint approximation. See also Jensen [4].

When we want to find an approximation for the distribution function, several saddlepoint methods are available. Most simple method is to integrate the approximated probability density function obtained by the saddlepoint method. However this integration may not be easily carried out. Another method is based on the inversion formula from the cumulant generating function to the distribution function. See Lugannani and Rice [5], and Daniels [2] for a review on the tail probability approximations.

Recall that the saddlepoint is defined by the solution T of the equation $\kappa'(T) = \bar{x}$, where $\kappa(T)$ is a cumulant generating function. The saddlepoint is useful for approximating the probability density function. In this article we consider the equation $\kappa'(T) = \bar{x} + 1/(nT)$ of T. Its solution will be called the quasi-saddlepoint. Using the quasi-saddlepoint, we propose an alternate approximation formula for the distribution function by evaluating the inversion formula (2.1).

§ 2. Approximation for the distribution function

Let X be a random variable with a distribution function $F(x)$. We denote its cumulant generating function by

$$\kappa(T) = \log \mathrm{E}\{\exp(TX)\} = \log\left(\int_{-\infty}^{\infty} \exp(Tx)\, dF(x)\right).$$

Suppose that $\kappa(T)$ is finite for $-a < T < b$, where a and b are positive constants. Our interest is to approximate the distribution function $\bar{F}^n(x)$ of the mean $\bar{X}_n$ of a sample of n independent observations from $F(x)$.