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Negative solutions of the generalized Liénard equation

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The aim of this paper is to state the conditions for the existence of negative solutions of the Liénard equation

$$x'' + f(x)x' + g(x) = 0. \tag{1}$$

Throughout this paper we will assume that

$$xf(x) > 0,\ xg(x) > 0 \qquad \text{for all } x \neq 0 \text{ and} \tag{2}$$

f and g are continuous on $R = (-\infty, \infty)$.

Under a negative solution $x(t)$ of (1) we will understand a solution such that $x(t) < 0$ on some interval $[T, \infty)$.

Denote

$$F(x) = \int_0^x f(s)ds, \quad G(x) = \int_0^x g(s)ds, \quad x \in (-\infty, \infty). \tag{3}$$

It follows from the assumptions (2) that $F(x) > 0$, $G(x) > 0$ for all $x \neq 0$, $F(0) = G(0) = 0$, $F(x)$ and $G(x)$ are decreasing for $x < 0$ and increasing for $x > 0$.

In the paper [1] we have made a qualitative analysis of the solutions of (1). In the paper [2] we have considered the behaviour and the existence of positive solutions of (1). In this paper we shall focus our attention on the negative solutions of (1). First, we shall introduce some results from [1] concerning the negative solutions of (1) and we shall use them later.

THEOREM A ([1], Theorem 4.1). *Let $x(t)$ be a solution of (1) such that $x(t_0) < 0$, $x'(t_0) \geqslant 0$. Then there exists $\tau > t_0$ such that $x(\tau) = 0$.*

COROLLARY 1. *Let $x(t) < 0$, $t > t_0$, be a solution of (1). Then $x'(t) < 0$ for $t \geqslant t_0$.*

THEOREM B ([1], Theorem 4.3). *Let $x(t) < 0$, $t \geqslant t_0$, be a solution of (1). Then $\lim x(t) = -\infty$ as $t \to \infty$.*

THEOREM C ([1], Theorem 4.5). *Suppose that $F(-\infty) < \infty$ and $\limsup g(x) < 0$ as $t \to -\infty$. Then the equation (1) has no negative solution.*