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Maximal conditions for locally finite Lie algebras and double chain conditions

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Introduction

A distinction between the minimal conditions for locally finite Lie algebras over a field of characteristic 0 has been exactly drawn by the author [5] and Stewart [7, 9]. However, very little is known concerning maximal conditions for Lie algebras. The first purpose of this paper is to distinguish between the maximal conditions for locally finite Lie algebras over a field of characteristic 0. The second one is to distinguish between various double chain conditions (i.e. maximal and minimal conditions) for Lie algebras.

In Section 2 we shall first prove that $\mathrm{L(wser)}\mathfrak{F} \cap \mathrm{Max}\text{-}\lhd = \mathfrak{F}$ over any field (Theorem 2.1), which is a generalization of [8, Theorem 6.5] and which suggests that under stronger conditions than local finiteness all the maximal conditions may be equivalent to each other. Secondly we shall prove that $\mathrm{L}\mathfrak{F} \cap \mathrm{Max}\text{-}\lhd^2 = \mathrm{L}\mathfrak{F} \cap \mathrm{Max}\text{-ser}$ over a field of characteristic 0 (Theorem 2.2) and shall exactly distinguish between the maximal conditions for locally finite Lie algebras over a field of characteristic 0 (Corollary 2.3).

In Section 3 we shall prove that $\mathrm{Max}\text{-}\lhd \cap \mathrm{Min}\text{-si} \leq \mathrm{Max}\text{-si}$ over any field (Theorem 3.2) and shall consequently draw a distinction between various double chain conditions for Lie algebras (Corollary 3.3). In addition, we shall prove that $\mathrm{Max}\text{-}\lhd^2 \leq \mathrm{Max}\text{-s}\mathfrak{F}$ over a field of characteristic 0 (Proposition 3.5), where $\mathrm{Max}\text{-s}\mathfrak{F}$ is the class of Lie algebras satisfying the maximal condition for finite-dimensional subideals.

In Section 4 we shall present a method of constructing Lie algebras satisfying neither the maximal condition nor the minimal condition for 2-step subideals and shall consequently prove that there exists a Lie algebra L over any field such that $L \in \mathrm{L}\mathfrak{F} \cap \mathrm{Max}\text{-}\lhd \cap \mathrm{Min}\text{-}\lhd$ and $L \notin \mathrm{Max}\text{-}\lhd^2 \cup \mathrm{Min}\text{-}\lhd^2$ (Theorem 4.6(1)).

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Throughout this paper we are always concerned with Lie algebras which are not necessarily finite-dimensional over an arbitrary field $\mathfrak{k}$ unless otherwise