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Behavior of bounded positive solutions of higher order differential equations

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1. Introduction

There is little known about the behavior of solutions of the differential equation of the form

$$(*) \qquad x^{(n)} + p(t)x^{(n-1)} + q(t)x^{(n-2)} + H(t, x) = 0$$

where $n \geq 3$ is an integer and $H: \mathfrak{R}^+ \times \mathfrak{R} \to \mathfrak{R}$ is continuous, decreasing in its second variable and is such that $uH(t, u) < 0$ for all $u \neq 0$. Some properties of solutions of $(*)$ are given by the author in [5] and [6]. In [7] the author gave two oscillation results for odd order equations with certain conditions on the functions p and q. This paper is a continuation of the study of differential equation $(*)$. Several results concerning bounded eventually positive solutions of $(*)$ will be proven. The nonlinear functionals which appear in the first two theorems can become very useful when studying the oscillatory behavior of solutions of $(*)$. This technique in fact was used in [7] as well as by Erbe [1], Heidel [2], Kartsatos [3], and Kartsatos & Kosmala [4].

2. Preliminaries

In what follows $\mathfrak{R}$ is used to denote the real line and $\mathfrak{R}^+$ the interval $(0, \infty)$. Also, $x(t)$, $t \in [t_x, \infty) \subset \mathfrak{R}^+$, is a solution of $(*)$ if it is n times continuously differentiable and satisfies $(*)$ on $[t_x, \infty)$. The number $t_x > 0$ depends on a particular solution $x(t)$ under consideration. We say that a function is "oscillatory" if it has an unbounded set of zeros. Moreover, a property P holds "eventually" or "for all large t" if there exists $T > 0$ such that P holds for all $t \geq T$. $C^n(I)$ denotes the space of all n times continuously differentiable functions $f: I \to \mathfrak{R}$. And we write $C(I)$ instead of $C^0(I)$. Throughout this paper we will assume that $p \in C^2[t_0, \infty)$, $q \in C^1[t_0, \infty)$ with

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