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Oscillation of differential equation of neutral type

Dedicated to Professor Takaŝi Kusano on his sixtieth birthday

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1. Introduction

Consider the neutral differential equation

$$\frac{d}{dt}[x(t) - p(t)x(\sigma(t))] + q(t)x(\tau(t)) = 0\,, \qquad t \geq t_0\,, \tag{1}$$

under the standing hypotheses that:

(a) $p \in C\ [[t_0, \infty), (0, \infty)]$;
(b) $\sigma \in C\ [[t_0, \infty), R]$, σ is strictly increasing and $\lim_{t\to\infty} \sigma(t) = \infty$;
(c) $q \in C\ [[t_0, \infty), R]$, $q(t) \not\equiv 0$;
(d) $\tau \in C\ [[t_0, \infty), R]$, $\lim_{t\to\infty} \tau(t) = \infty$.

Our aim in this paper is to obtain sufficient conditions for the oscillation of all solutions of equation (1). The asymptotic behaviour of the solutions of equation (1) is also studied.

By a solution of equation (1) we mean a continuous function $x: [T_x, \infty) \to R$ such that $x(t) - p(t)x(\sigma(t))$ is continuously differentiable and $x(t)$ satisfies equation (1) for all sufficiently large $t > T_x$. The solutions which vanish for all large t will be excluded from our consideration. A solution of (1) is said to be oscillatory if it has an infinite sequence of zeros tending to infinity; otherwise a solution is said to be nonoscillatory.

The problem of oscillation and nonoscillation for neutral differential equations has received considerable attention in recent years; see e.g. [1–7, 9] and the references cited therein. However some results in this paper are new and the other ones in many cases complete the previous ones.

2. Some basic lemmas

The following lemmas will be useful in the proofs of the main results.

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