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Difference families with applications to resolvable designs

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Abstract. Some block disjoint difference families are constructed in rings with the property that there are k distinct units u_i, $0 \le i \le k-1$, such that differences $u_i - u_j$ $(0 \le i < j \le k-1)$ are all units. These constructions are utilized to produce a large number of classes of resolvable block designs.

1. Introduction

A *balanced incomplete block design* (or, *design*) $B(k, \lambda; v)$ is a pair $(\mathscr{V}, \mathscr{B})$ where $\mathscr{V}$ is a set of v *points* (called *treatments*), and $\mathscr{B}$ is a collection of subsets (called *blocks*) of $\mathscr{V}$, each of size k, such that every pair of distinct points from $\mathscr{V}$ is contained in exactly λ blocks. Note that λ is called the *index*.

One way of investigating the structure of a design is to look at its "symmetry", which can be formalized as the automorphism group of the design. Let $(\mathscr{V}, \mathscr{B})$ be a design and let $\phi: \mathscr{V} \to \mathscr{V}$ be a bijection. The mapping Φ *induced by* ϕ has domain $\mathscr{B}$ and is defined by $\Phi(B) = \{\phi(x): x \in B\}$. An *automorphism* of the design $(\mathscr{V}, \mathscr{B})$ is a pair of bijections $\phi: \mathscr{V} \to \mathscr{V}$ and $\psi: \mathscr{B} \to \mathscr{B}$ which preserves incidence, that is, $\psi(B) = \Phi(B)$ for all $B \in \mathscr{B}$. The set of all automorphisms of $(\mathscr{V}, \mathscr{B})$ forms a group under composition called the *automorphism group* of the design.

Let G be an additive abelian group and $B = \{b_1, \ldots, b_k\}$ be a subset of G. Define the *development* of B as

$$\operatorname{dev} B = \{B + g: g \in G\},$$

where $B + g = \{b_1 + g, \ldots, b_k + g\}$ for $g \in G$.

Let $\mathscr{F} = \{B_1, \ldots, B_t\}$ be a family of subsets of G and define the *development* of $\mathscr{F}$ as

$$\operatorname{dev} \mathscr{F} = \bigcup_{i=1}^{t} \operatorname{dev} B_i.$$

If $\operatorname{dev} \mathscr{F}$ is a $B(k, \lambda; v)$, it is said that $\mathscr{F}$ is a $(k, \lambda; v)$ *difference family*, denoted by $DF(k, \lambda; v)$, and the sets $B_1, \ldots, B_t$ are called *base blocks* (or *initial blocks*). The group G is contained in the automorphism group of $\operatorname{dev} \mathscr{F}$.