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Global real analytic length parameters for Teichmüller spaces

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ABSTRACT. It is well-known that Teichmüller space is a global real analytic manifold. Using the geometry of Möbius transformations and the one-half power of a hyperbolic transformation, we consider the minimal number of global real analytic length parameters for Teichmüller space and such length parameter space.

1. Introduction

A Riemann surface S of genus g with m holes is called of *type* $(g, 0, m)$. If $2g + m \geq 3$, then S is conformally equivalent to the quotient space $\boldsymbol{D}/G$, where $\boldsymbol{D}$ is the unit disk in the complex plane and G is a Fuchsian group acting on $\boldsymbol{D}$. This G is also called of *type* $(g, 0, m)$. Teichmüller space $T(g, 0, m)$, $2g + m \geq 3$ is the set of equivalence classes of marked Fuchsian groups of type $(g, 0, m)$ and a global real analytic manifold of dimension $6g + 3m - 6$. It is well known that $T(g, 0, m)$ is parametrized global real analytically by some lengths of closed geodesics on a Riemann surface represented by a marked Fuchsian group (see for example, [1], [4], [6], [7], [8], [13] and [16]). Such lengths are called *length parameters*. In this paper, we consider the following problem.

PROBLEM. (i) What is the minimal number of global real analytic length parameters for $T(g, 0, 0)$, $g \geq 2$?
(ii) How is the parameter space described by such length parameters?

About the first problem, Wolpert [20] and [21] announced that the minimal number of these parameters is greater than $\dim(T(g, 0, 0)) = 6g - 6$. Next, Seppälä and Sorvali [17] and Okumura [8] showed that this minimal number is less than or equal to $6g - 4$. Finally, we concluded that *the minimal number of global real analytic length parameters for* $T(g, 0, 0)$, $g \geq 2$ *is* $6g - 5$ *and, further, that we can take these lengths from simple closed geodesics on a Riemann surface.* This was first proved by Schmutz [14]. In the

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