

Stable and unstable sets for evolution equations of parabolic and hyperbolic type

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ABSTRACT. Some roles in the global dynamics of so called stable and unstable sets will be given for semilinear heat equations and semilinear wave equations with dissipative terms.

1. Introduction

Let $\Omega \subset R^N$ be a bounded domain with smooth boundary $\partial\Omega$. We are concerned with the following two mixed problems:

$$(1.1) \quad u_t - \Delta u = |u|^{p-1}u, \quad x \in \Omega, t > 0,$$

$$(1.2) \quad u(0, x) = u_0(x), \quad x \in \Omega,$$

$$(1.3) \quad u(t, x)|_{\partial\Omega} = 0 \quad \text{for } t \geq 0,$$

and

$$(1.4) \quad u_{tt} - \Delta u + \delta u_t = |u|^{p-1}u, \quad x \in \Omega, t > 0,$$

$$(1.5) \quad u(0, x) = u_0(x), \quad u_t(0, x) = u_1(x), \quad x \in \Omega,$$

$$(1.6) \quad u(t, x)|_{\partial\Omega} = 0 \quad \text{for } t \geq 0.$$

Here $p > 1$, $\delta \geq 0$ and Δ is the Laplacian in R^N .

For these problems, many authors investigated their dynamics. In particular, since Sattinger [21] has constructed so called stable set in 1968, the method of stable set (potential well) was used in order to construct global solutions (Ebihara et al. [3], Ikehata [9], Ishii [11], Lions [14], Nakao et al. [16], Ōtani [17] and Tsutsumi [22, 23] e.g.). Furthermore, with respect to the blowing-up properties, there is a work of Payne et al. [19]. Namely, roughly speaking, if initial data u_0 belongs to so called unstable set, then the associated weak solution blows up in a finite time. Of related interest is the works of Ikehata et al. [10], Ishii [11] and Ōtani [17, 18].

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