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Foliations and divergences of flat statistical manifolds

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ABSTRACT. A Hessian domain $(\Omega, \tilde{D}, \tilde{g} = \tilde{D}d\varphi)$ is a flat statistical manifold, and level surfaces of φ are 1-conformally flat statistical submanifolds of $(\Omega, \tilde{D}, \tilde{g})$. In this paper we consider a foliation defined by level surfaces of φ and its orthogonal foliation, and then we investigate divergences restricted to leaves of these foliations.

1. Introduction

Statistical manifolds have been studied in terms of information geometry. *Dualistic structures* of statistical manifolds play important roles on statistical inference, control systems theory, and so on [1] [12]. It is known that a *Hessian structure* is a *dually flat structure* and gives, for examples geometry of an exponential family [14]. Applications of the dually flat structures of submanifolds are in [4] [12]. Non-flat statistical manifolds are studied in [6] [7] [8]. It seems that there are not results on *statistical submanifolds* without dually flat structures. So, we treat non-flat dualistic structures on submanifolds, especially on *level surfaces* of *Hessian domain*, and show 1-conformal flatness, if considering a Hessian domain as a flat statistical manifold.

Let φ be a function on a domain Ω in a real affine space $\mathbf{A}^{n+1}$. Denoting by $\tilde{D}$ the canonical flat affine connection on $\mathbf{A}^{n+1}$, we set $\tilde{g} = \tilde{D}d\varphi$ and suppose that $\tilde{g}$ is non-degenerate. Then a Hessian domain $(\Omega, \tilde{D}, \tilde{g})$ is a *flat statistical manifold*. In [15] we proved that n-dimensional level surfaces of φ are *1-conformally flat statistical submanifolds* of $(\Omega, \tilde{D}, \tilde{g})$. Using this fact, we show that *dual-projectively equivalent* affine connections can be led on a leaf of a foliation $\mathscr{F}$ defined by n-dimensional level surfaces of φ on Ω. In addition we study the *orthogonal foliation* $\mathscr{F}^{\perp}$ of $\mathscr{F}$.

We also discuss *divergences* on leaves of the foliations $\mathscr{F}$ and $\mathscr{F}^{\perp}$ in §4. Nagaoka and Amari first studied divergences of flat statistical manifolds in view of statistics [1]. Kurose defined the canonical divergences of 1-conformally flat statistical manifolds [7]. In this paper we show that, for $M \in \mathscr{F}$, Kurose's

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