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Asymptotic expansions of the null distributions of three test statistics in a nonnormal GMANOVA model

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ABSTRACT. This paper deals with three test statistics for testing a linear hypothesis and estimators of regression coefficients in the GMANOVA model which was proposed by Potthof and Roy (1964), without assuming normal error. The test statistics considered include the likelihood ratio statistic, the Lawley-Hotelling trace criterion and the Bartlett-Nanda-Pillai trace criterion, which have been proposed under normality. We obtain asymptotic expansions of the null distributions of three test statistics up to the order n^{-1}, where n is the sample size. The results are generalizations of the formulas in Wakaki, Yanagihara and Fujikoshi (2000). In addition, asymptotic expansions of the distribution functions of several standardized statistics on regression coefficients are derived.

1. Introduction

The GMANOVA model considered is defined by

$$Y = A\Xi X' + \mathscr{E}, \tag{1.1}$$

where $Y = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_n)'$ is an $n \times p$ observation matrix of response variables, $A = (\boldsymbol{a}_1, \ldots, \boldsymbol{a}_n)'$ is an $n \times k$ between-individuals design matrix of explanatory variables with full rank k, X is a $p \times q$ within-individuals design matrix of explanatory variables with full rank q $(\leq p)$, Ξ is a $k \times q$ unknown parameter matrix and $\mathscr{E} = (\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_n)'$ is an $n \times p$ error matrix. It is assumed that each vector $\boldsymbol{\varepsilon}_j$ is *i.i.d.*, i.e., independently and identically distributed with $\mathrm{E}(\boldsymbol{\varepsilon}_j) = \boldsymbol{0}$ and $\mathrm{Cov}(\boldsymbol{\varepsilon}_j) = \Sigma$. This model can be applied to analysis of growth curve data, and hence it is also called the growth curve model.

We consider to test for a general linear hypothesis

$$H_0 : C\Xi D = O, \tag{1.2}$$

where C is a known $c \times k$ matrix with rank c $(\leq k)$, D is a known $q \times d$ matrix with rank d $(\leq q)$ and O is a $c \times d$ matrix all of whose elements are 0.

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