

Open books on 5-dimensional manifolds

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(Received May 9, 2001)

ABSTRACT. We first give a necessary and sufficient condition for two open books on a rational homology 5-sphere to have equivalent bindings, which generalizes a result in [22] for open books on S^5 . Second we give an existence theorem of an open book realizing a given 4-manifold as its page, up to taking connected sum with some copies of $S^2 \times S^2$. Finally, as an application of all these results, we give an algebraic criterion for two diffeomorphisms of a compact simply connected 4-manifold with boundary to be isotopic up to connected sum with some copies of $S^2 \times S^2$, which is similar to a result of Quinn [19] for closed 4-manifolds.

1. Introduction

Let M be a smooth closed manifold and K a codimension two submanifold with trivial normal bundle. We assume that the complement $M \setminus K$ admits a smooth locally trivial fibration ϕ over S^1 which is consistent with the normal bundle of K . Then the triad (M, K, ϕ) is called an *open book* (for details, see Definition 2.1). Furthermore, K is called the *binding*, and the closure in M of each fiber of the fibration is called a *page*, which can be regarded as a Seifert manifold of K .

Open book structures on odd dimensional manifolds have been extensively studied and various important results have been obtained (for example, see [5], [6], [12], [17], [25], [27]). However, almost all the results have concerned open books on manifolds whose dimensions are greater than or equal to 7. It has been observed that, even when the ambient manifold M is diffeomorphic to the sphere, the situation is totally different for dimension 5 (for details, see [20], [21], [22]).

When $M = S^5$, many results have been obtained in [20], [21], [22]. In this paper, we generalize some of the results in [22] to open books on 1-connected 5-dimensional manifolds which are not necessarily spheres. We note that in higher dimensions, a classification of certain open books on highly connected manifolds which are not necessarily spheres has been obtained in [15].

2000 *Mathematics Subject Classification.* Primary 57Q45; Secondary 57R52.

Key words and phrases. open book, rational Seifert form, fibered knot, isotopy, 4-manifold with boundary, variation map.

The author has been supported in part by Grant-in-Aid for Scientific Research (No. 13640076), Ministry of Education, Science and Culture, Japan.