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Correspondences to abelian varieties II

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Abstract. When S is an algebraic scheme, and $X \to S$ and $Y \to S$ proper schemes over S, we define the notion of correspondences from X to Y over S. And when $Y \to S$ is a relative abelian scheme and X is a normal variety, we give a characterization for a correspondence from X to Y over S to be a graph of some morphism $X \to Y$ over S, which is a generalization of the result for classical correspondences in [4].

1. Introduction

Let $\alpha : X \vdash Y$ be a correspondence over a point, i.e., an element of the Chow group of $X \times Y$ where X and Y are smooth complete algebraic varieties [1, Chapter 16]. When $\alpha = \Gamma_f$ is a graph of some morphism $f : X \to Y$, then α satisfies the following 3 conditions:

(1) $\dim(\alpha) = \dim(X)$

(2) $\pi_*(\alpha) = [X]$

(3) $\Delta_Y \circ \alpha = (\alpha \times \alpha) \circ \Delta_X$.

Conversely, if the conditions (1), (2) and (3) are satisfied and Y is an Abelian variety, then α is a graph of some morphism [4, Theorem 2.7]. In the paper [5], the notion of correspondences is generalized to the situation where the base scheme can be any algebraic scheme, as far as the structure morphism is proper. In this paper, we will show that the result of [4] is valid in this general case, namely when Y is a relative abelian scheme over an algebraic scheme S, X is scheme over S with the structure morphism proper, and α a correspondence over S.

Convention and Notation. We work in the category of algebraic schemes over a fixed field κ. A variety means a reduced and irreducible scheme. A scheme X is smooth when the structure morphism $X \to \operatorname{Spec} \kappa$ is smooth. A_*X is the Chow group of X and A^*X the Chow cohomogloy group (see [1, Chapter 17]), with rational coefficients. The bivariant groups also have rational coefficients. The notation $X \overset{@}{\to} Y$ means that there exists a morphism $X \to Y$ and an element of the bivariant intersection group $\alpha \in A(X \to Y)$.

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