HIROSHIMA MATH. J.
11 (1981), 285–297

Generally induced modules in Lie algebras

Fujio KUBO

(Received September 19, 1980)

Introduction

Throughout this paper let K denote a field of characteristic zero and $U(M)$ the enveloping algebra of a Lie algebra M over K. Now let G be a Lie algebra over K. Let H be a subalgebra of G and W be an H-module. Regarding $U(G)$ as a right $U(H)$-module, we can form the left G-module $U(G)\otimes_{U(H)} W$. This module is called the G-module induced by W and discussed in [1, pp. 169–189].

In this paper we generalize the construction of the induced G-module to define the generally induced G-module by taking a subalgebra R of $U(G)$ instead of taking a subalgebra of G. We mainly investigate the generally induced G-module in the case that $U(G)$ has a good basis, namely a regular basis, as a right R-module.

For $u \in U(G)$ we say that u is permutable with R if $Ru = uR$. Then we have an automorphism $p(u)$ of R such that $ru = up(u)(r)$ for any $r \in R$. We call it the permuting map of R associated with u. The permuting map will play an important role to investigate the generally induced module.

In §3 we give several conditions under which every R-endomorphism of an irreducible R-module W is algebraic over K. Such conditions enable us to have a central character of the R-module W when K is algebraically closed. We then give criteria of the homogeneity of an R-submodule of $U(G)\otimes_R W$ by using the central character and the permuting map of R given in §2.

In §4 we discuss the structure and the classification of $R[u_\lambda, u_\tau]\otimes_R W$ in the case that $u_\lambda u_\tau \in R$, where u_λ and u_τ belong to a regular basis of $U(G)$, $u_\lambda \neq 1$ and $u_\tau \neq 1$.

In §5 and §6 we apply the results given in §§1–4 to the case that G is $sl(2, K)$ or the Heisenberg algebra. Generally induced modules given in these sections cannot be constructed as any modules induced by modules over their proper subalgebras.

§1. Definition of generally induced modules

DEFINITION. Let G be a Lie algebra over K and R be a subalgebra of $U(G)$. For an R-module W we can form the left G-module

$$U(G) \otimes_R W,$$