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MINIMAL SUBMANIFOLDS OF ALMOST SEMI-KÄHLER MANIFOLDS

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The purpose of this note is to study the minimality of almost Hermitian submanifolds of codimension 2 in an almost semi-Kähler manifold. Our main result is Theorem 3.4.

In § 1, we review the various classes of almost Hermitian manifolds, and prove, in passing, that all four-dimensional almost semi-Kähler manifolds are almost Kähler. In this manner, the inclusion lattice of almost Hermitian structures is greatly reduced in the four-dimensional case, (Figure 2). In § 2, we recall the configuration tensor of an immersed Riemannian submanifold and its relation to the second fundamental form of the immersion. In § 3, we study the effect of certain almost Hermitian structures on the configuration tensor, proving the minimality of codimension 2 almost Hermitian submanifolds of an almost semi-Kähler manifold.

§ 1. Almost Semi-Kähler Manifolds.

Let (M, g, F) be a smooth, *almost Hermitian manifold.* That is, M is a smooth, connected paracompact manifold; F is a smooth tensor field of type (1, 1) satisfying $F^2=-1$; g is a smooth Riemannian structure on M; and the tensors F and g satisfy:

$$g(X, Y)=g(FX, FY),$$

for all smooth vector fields, X and Y on M. The *Kähler form* on M is the differential 2-form of bidegree (1, 1) given by:

$$\Omega(X, Y)=g(X, FY).$$

An almost Hermitian manifold is necessarily orientable and of even dimension, which we shall take to be $2n$.

The manifold (M, g, F) is said to be *almost semi-Kähler* if Ω if coclosed; i. e., if its codifferential, $\delta\Omega$, vanishes. An almost semi-Kähler space is said to be *semi-Kähler* if it is complex, or, equivalently, if the almost complex structure, F, is integrable. Other classes of almost Hermitian manifolds are defined as follows. We say that (M, g, F) is:

($\mathcal{QK}$) *Quasi-Kähler* if the components of bidegree (1, 2) and (2, 1) of the 3-form

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