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SOME CONDITIONS FOR CONSTANCY OF THE HOLOMORPHIC SECTIONAL CURVATURE

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1. Introduction. Let M be a Kaehler manifold with complex structure J and Riemann metric $\langle\ ,\ \rangle$. By a plane section σ we mean a 2-dimensional linear subspace of a tangent space of M. If σ is invariant under the action of J, it is called *holomorphic section*. If σ is perpendicular to $J\sigma$, it is called *anti-holomorphic section*. A Kaehler manifold of constant holomorphic sectional curvature is called a *complex space form*. Then the following result is known.

THEOREM A (B. Y. Chen and K. Ogiue). *Let M be a Kaehler manifold. If the anti-holomorphic sectional curvatures of M are constant and if* $\dim M \geqq 3$, *then M is a complex space form.*

In Section 2, we shall define the angle between two linear subspaces of a vector space. Then we shall call a plane section σ a *θ-holomorphic section* if the angle between σ and $J\sigma$ is θ and the sectional curvature for a θ-holomorphic section the *θ-holomorphic sectional curvature*. It is clear that a complex space form has constant θ-holomorphic sectional curvatures. Conversely, in Section 3, we shall prove

THEOREM 3. *Let M be a Kaehler manifold. If the θ-holomorphic sectional curvatures of M are constant and if* $\cos\theta \neq 0$, *then M is a complex space form.*

Next, the holomorphic bisectional curvature $H(\sigma, \sigma')$ for holomorphic planes σ and σ' is defined by S. I. Goldberg and S. Kobayashi ([2]) as $H(\sigma, \sigma') = \langle R(X, JX)JY, Y\rangle$, where R is the curvature tensor of M and X, Y are unit vector in σ and σ' respectively. We shall call $H(\sigma, \sigma')$ a *holomorphic τ-bisectional curvature* if the angle between σ and σ' is τ. Then it is clear that a complex space form has constant holomorphic τ-bisectional curvatures. We shall prove

THEOREM 4. *Let M be a Kaehler manifold. If the holomorphic τ-bisectional curvatures of M are canstant and if $\tau \neq \frac{\pi}{2}$, then M is a complex space form.*

THEOREM 5. *Let M be a Kaehler manifold. If the holomorphic $\frac{\pi}{2}$-bisectional curvatures of M are constant and if* $\dim M \geqq 3$, *then M is a complex space form.*

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