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ON AUTOMORPHISM GROUPS OF QUATERNION KÄHLER MANIFOLDS

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It is a well-known result that the group of isometries $I(M)$ of an n-dimensional Riemannian manifold M is of dimension at most $\frac{1}{2}n(n+1)$. And if $\dim I(M)=\frac{1}{2}n(n+1)$, then M is isometric to one of the following spaces of constant curvature: (a) an n-dimensional Euclidean space R^n; (b) an n-dimensional sphere S^n; (c) an n-dimensional projective space $Pn(R)$; (d) an n-dimensional simply connected hyperbolic space. In 1947, Wang [11] showed that the group of isometries of an n-dimensional Riemannian manifold with $n\neq 4$ has no closed subgroup of dimension r for $\frac{1}{2}n(n-1)+1<r<\frac{1}{2}n(n+1)$ (See also Yano [13]). And in 1954, Ishihara [5] proved that in a Kähler manifold M the group of automorphisms $A(M)$ of a $2m$-dimensional Kähler manifold M with $m\geqq 3$, $m\neq 4$ contains no closed subgroup of dimension r for $m^2+2<r<m^2+2m-1$. On the other hand, recently, quaternion Kähler manifolds have been studied by several authors (Alekseevski [1], [2], Gray [4], Ishihara [6], [7] Ishihara and Konishi [8] and Wolf [12]). The purpose of this paper is to prove for quaternion Kähler manifolds a theorem stated in the last part of § 5 which is similar to the Wang's theorem for Riemannian case. If M is a $4m$-dimensional quaternion Kähler manifold, then the maximum dimension of the automorphism group $A(M)$ is $2m^2+5m+3$, as will be seen in Lemma 2.1. And it is known that if the maximum dimension of the automorphism group is attained, i. e., the isotropy subgroup is $Sp(m)\cdot Sp(1)=Sp(m)\times Sp(1)/\{\pm 1\}$, then M is isomorphic to one of the following spaces: (a) a $4m$-dimensional Euclidean space Q^m; (b) a quaternion projective space $P^m(Q)$; (c) a quaternion hyperbolic space form [2].

In § 1 and § 2, we recall definitions and some properties of quaternion Kähler manifolds and its automorphisms. In § 3, we recall some algebraic lemmas for later use. § 4 and § 5 are devoted to prove our main results which will be stated in § 5. Manifolds, mappings, tensor fields and other geometric objects we discuss are assumed to be differentiable and of class C^∞.

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