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METRIC POLYNOMIAL STRUCTURES

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0. The paper is devoted to the study of metric polynomial structures, i. e. polynomial structures f for which there exists a positive definite Riemannian metric g such that there is $g(f(X), f(Y))=g(X, Y)$. In the first paragraph we divide the metric polynomial structures into four groups, restricting then ourselves to the first group only. In the second paragraph we are concerned with the integrability conditions of the distributions naturally arising in the study of these structures. In the last third paragraph we establish the existence of special connections associated with the metric polynomial structures.

1. Let M be a differentiable manifold of class C^{∞}. By a *polynomial structure* on M we mean a C^{∞}-tensor field of type (1, 1) on M satisfying a polynomial equation

$$P(f)=f^{n}+a_{1}f^{n-1}+\cdots+a_{n-1}f+a_{n}I=0$$

where $a_1, \cdots, a_n$ are real numbers, at every point of M. Moreover we shall suppose that the polynomial P is the minimal polynomial of f_x at every point $x\in M$.

Example: If f satisfies a polynomial equation $P(f)=0$ then P need not be necessarily the minimal polynomial of f_x at every point $x\in M$, even if we suppose that f has a constant rank on M. Let us take for example $M=\boldsymbol{R}^4$ with cartesian coordinates (X_1, X_2, X_3, X_4) and let us define f by

$$f=\begin{pmatrix} 0 & x_1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

It is easy to see that f satisfies on $\boldsymbol{R}^4$ the equation $P(\xi)=\xi^3=0$. Its minimal polynomial at a point with $x_1\neq 0$ is ξ^3 whereas it is ξ^2 at a point with $x_1=0$. Clearly rank $f=2$ on M.

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