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KŌDAI MATH. SEM. REP.
27 (1976), 188—192

THE AXIOM OF SPHERES IN KAEHLER GEOMETRY

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1. Introduction. Let M be an Hermitian manifold of complex dimension >1 with almost complex structure J and Riemannian metric g. A 2-dimensional subspace σ of M_m, the tangent space of M at m, is called a *holomorphic* (resp., *antiholomorphic*) *plane* if $J\sigma=\sigma$ (resp., $J\sigma$ is orthogonal to σ). M is said to satisfy the *axiom of holomorphic* (resp., *antiholomorphic*) *planes* if for every $m\in M$ and every holomorphic (resp., *antiholomorphic*) *plane* σ at m, there exists a totally geodesic submanifold N satisfying $m\in N$ and $N_m=\sigma$. Yano and Mogi [7] showed that a Kaehler manifold satisfying the axiom of holomorphic planes has constant holomorphic curvature. The same conclusion prevails for a Kaehler manifold satisfying the axiom of antiholomorphic planes, as was recently shown by Chen and Ogiue [2].

A Riemannian manifold M of (real) dimension $d\geqq 3$ is said to satisfy the *axiom of r-spheres* ($2\leqq r<d$) if for each $m\in M$ and any r-dimensional subspace S of M_m, there exists an r-dimensional umbilical submanifold N with parallel mean curvature vector field satisfying $m\in N$ and $N_m=S$. This notion was introduced by Leung and Nomizu [6] who proved that a manifold with this property for some fixed r, $2\leqq r<d$, has constant sectional curvature. This generalizes the well-known theorem of Cartan [1] concerning the axiom of r-planes.

For an Hermitian manifold M, one of the authors [3] recently introduced the *axiom of holomorphic 2-spheres* and generalized the theorem of Yano and Mogi. Similarly, Harada [5] has introduced the *axiom of antiholomorphic 2-spheres* and generalized the theorem of Chen and Ogiue.

A subspace S of M_m, where M is an Hermitian manifold, is said to be *holomorphic* (resp., *antiholomorphic*) if $JS=S$ (resp., JS is orthogonal to S). Let $d=\dim_C M$.

Axiom of holomorphic $2r$-planes (resp., **$2r$-spheres**). *For each $m\in M$ and $2r$-dimensional holomorphic subspace S of M_m, $1\leqq r<d$, there exists a totally geodesic submanifold* (resp., *umbilical submanifold with parallel mean curvature vector field*) *N satisfying $m\in N$ and $N_m=S$.*

Axiom of antiholomorphic r-planes (resp., **r-spheres**). *For each $m\in M$ and r-dimensional antiholomorphic subspace S of M_m, $2\leqq r<d$, there exists a totally geodesic submanifold* (resp., *umbilical submanifold with parallel mean curvature*

Received April 10, 1974.

1) Research partially supported by the National Science Foundation.