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SINGULAR SETS OF SOME INFINITELY GENERATED KLEINIAN GROUPS

Dedicated to Professor Yûsaku Komatu on his 60th birthday

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§1. Preliminaries and Notations.

1. Let $\{K_j\}_{j=1}^{p}$ and $\{H_i, H_i'\}_{i=p+1}^{\infty}$ be an infinite number of circles external to one another in the extended complex plane $\tilde{C}=\{z;\ |z|\leqq\infty\}$, where $\{H_i, H_i'\}_{i=p+1}^{q}$ tend to only a finite point Q for $q\to\infty$. Let B be a domain bounded by these circles. Without loss of generality we may assume that these circles are contained in some closed disc $D_0=\{z;\ |z|\leqq\rho_0\}$.

Let $\{T_j\}_{j=1}^{p}$ be the elliptic transformations with period 2 corresponding to $\{K_j\}_{j=1}^{p}$, each of which transforms the outside of K_j onto the inside of itself. Let $\{T_i\}_{i=p+1}^{\infty}$ be the system of hyperbolic or loxodromic transformations, each T_i of which transforms the outside of H_i' onto the inside of H_i. Then the system $\mathfrak{G}=\{T_i, T_i^{-1}\}_{i=1}^{\infty}$ $(T_i=T_i^{-1},\ 1\leqq i\leqq p)$ generates an infinitely generated discontinuous group denoted by G and we call $\mathfrak{G}$ the generator system of G, where T_i^{-1} denotes the inverse of T_i.

The purpose of this paper is to investigate the singular set E of G. Take a positive integer q $(>p)$ and consider a subset $\mathfrak{G}_N=\{T_j\}_{j=1}^{p}\cup\{T_i, T_i^{-1}\}_{i=p+1}^{q}$ $(N=2q-p)$ of $\mathfrak{G}$. Then $\mathfrak{G}_N$ generates a finitely generated subgroup G_N of G. If we denote by B_N a domain bounded by $\{K_j\}_{j=1}^{p}\cup\{H_i, H_i'\}_{i=p+1}^{q}$ $(N=2q-p)$, it is well known that B_N coincides with a fundamental domain of G_N. We gave some results with respect to the singular set E_N of G_N by using the relations between E_N and the computing functions on G_N ([1]). We shall get G from G_N for $N\to\infty$. It is natural to try whether we can extend the results for G_N to ones for G. Unfortunately, we can not extend those in the same way, for the behavior of the accumulation of the circles to Q gives the complicated difficulty. Therefore we must impose some restrictions with respect to the accumulation of circles and henceforth we shall consider only such groups.

2. Denote by $r(H)$ the radius of a circle $H\in\{H_i, H_i'\}_{i=p+1}^{\infty}$ and assume that there exists some positive constant K independent of H such that it holds

$$\text{(A)}\qquad \frac{r(H)}{l(H)}\leqq K,$$

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