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ON THE EXISTENCE OF ANALYTIC MAPPINGS

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1. Let R be an ultrahyperelliptic surface defined by $y^2=G(x)$ where $G(x)$ is an entire function having only an infinite number of simple zeros. Let S be a similar surface defined by $y^2=g(x)$ with a similar $g(x)$ as $G(x)$. We already discussed the existence problem of non-trivial analytic mappings of R into S [4]. The following theorem is central theorem in the existence problem of non-trival analytic mappings.

THEOREM A. *Suppose that there is a non-trivial analytic mapping of R into S. Then there are entire $h(z)$ and meromorphic $f(z)$ satisfying $g\circ h(z)=f^2(z)G(z)$ and vice versa.*

Hiromi and Mutō [2] proved the following

THEOREM B. *Suppose that the order of $N(r, 0, G)$ is finite and the one of $N(r, 0, G)$ is finite positive and that there is a non-trivial analytic mapping of R into S. Then the corresponding $h(z)$ is a polynomial of degree* $\mathrm{ord}\, N(r, 0, G)/\mathrm{ord}\, N(r, 0, g)$.

In this paper we shall prove the following

THEOREM 1. *Suppose that the assumptions of Theorem* B *are satisfied and further that G is an entire periodic function of finite order. Then the existence of a non-trivial analytic mapping of R into S gives the following relation*

$$\mathrm{ord}\, G=\nu\, \mathrm{ord}\ N(r, 0, g) \qquad \nu=\begin{cases} 1, 2 & \text{if } \mathrm{ord}\, G\neq 2, \\ 1, 2, 3, 4, 6 & \text{if } \mathrm{ord}\, G=2. \end{cases}$$

There are examples which show the occurence of all the possible cases. In order to prove the above theorem we need several lemmas on number theory.

We shall give an application of Theorem 1.

THEOREM 2. *Besides the assumptions of Theorem* 1 *assume that g is also periodic. Then every non-trivial analytic mapping of R into S reduces to a conformal mapping of R onto S.*

Our results do not depend on any representations of R and S. We can for-

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