ISHIHARA, S. AND M. KONISHI
KŌDAI MATH. SEM. REP
25 (1873), 175—189

FIBRED RIEMANNIAN SPACE WITH TRIPLE OF KILLING VECTORS

BY SHIGERU ISHIHARA AND MARIKO KONISHI

Dedicated to Professor Shigeo Sasaki on his sixtieth birthday

Recently, 3-structures, almost contact, K-contact or Sasakian (normal contact), have been introduced and several interesting subjects concerning these structures have been studied ([3], [4], [5], [6], [8], [9], [13]). The 3-structure, K-contact or Sasakian, is a special kind of triples of Killing vectors, which will be defined in the present paper as a set of three unit Killing vectors ξ, η and ζ being mutually orthogonal and satisfying the structure equations $[\eta, \zeta]=2\xi$, $[\zeta, \xi]=2\eta$, $[\xi, \eta]=2\zeta$. One of purposes of the present paper is to obtain, in terms of curvatures, a condition that a triple of Killing vectors is a Sasakian 3-structure.

In §1, we recall definitions and properties of structures, K-contact or Sasakian. We define also in §1 a triple of Killing vectors and give its preliminary properties. In §2, we give fundamental concepts and divices concerning fibred Riemannian spaces with triple of Killing vectors. We state, in §3, some propositions concerning triples of Killing vectors or K-contact 3-structures as consequences of formulas established in §2. The last §4 is devoted to studying properties of Nijenhuis tensor of structure tensor fields determined by a triple of Killing vectors or a K-contact 3-structure.

§1. Preliminaries.

First, we recall some properties of a K-contact structure. Let $(\tilde{M}, \tilde{g})$ be a Riemannian manifold[1)] of dimension n with metric tensor $\tilde{g}$. Let there be given in $(\tilde{M}, \tilde{g})$ a unit Killing vector ξ satisfying

$$\tilde{K}(\xi, \tilde{X})\xi = -\tilde{X} + \alpha(\tilde{X})\xi, \tag{1.1}$$

where $\tilde{K}$ denotes the curvature tensor of $(\tilde{M}, \tilde{g})$ and α the 1-form associated with ξ, i.e., $\alpha(\tilde{X}) = \tilde{g}(\xi, \tilde{X})$.[2)] Then ξ is said to define a *K-contact structure* (cf. [2]). If we put, for a K-contact structure ξ,

Received May 8, 1972.

1) Manifolds, vector fields and geometric objects we discuss are assumed to be differentiable and of class C^∞.

2) Here and in the sequel, $\tilde{X}$, $\tilde{Y}$ and $\tilde{Z}$ denote arbitrary vector fields in $\tilde{M}$